Modified Cam Clay (MCC) Model

The Original Cam-Clay model is one type of CSSM model and is based on the assumption that the soil is isotropic, elasto-plastic, deforms as a continuum, and it is not affected by creep. The yield surface of the Cam clay model is described by a log arc.

Modified Cam-Clay Model
Professor John Burland was responsible for the modification to the original model, the difference between the Cam Clay and the Modified Cam Clay (MCC) is that the yield surface of the MCC is described by an ellipse and therefore the plastic strain increment vector (which is vertical to the yield surface) for the largest value of the mean effective stress is horizontal, and hence no incremental deviatoric plastic strain takes place for a change in mean effective stress.

- Explains the pressure-dependent soil strength and the volume change (contraction and dilation) of clayey soils during shear.
- When critical state is reached, then unlimited soil deformations occur without changes in effective stress or volume.
- Formulation of the modified Cam clay model is based on plastic theory which makes it possible to predict volume change due to various types of loading using an associated flow rule

Critical State

Applying shear stress to a soil will eventually lead to a state where no volume change occurs as the soil is continually sheared. When this condition is reached, it is known as the critical state.
Critical state and normally consolidated lines in $p'$-q-e space.

Note that the NC line falls on the $e$ vs $p'$ plane because no shear stress is present.

Note that the critical state line will parallel the NC line if this line is projected onto the $e$ vs $p'$ plane and is transformed to $e$ vs. $\ln p'$.
State Variables
- Mean effective stress, $p'$
- Shear stress, $q$
- Void ratio

Mean effective stress

$$p' = \frac{\sigma_1' + \sigma_2' + \sigma_3'}{3}$$

Shear stress

$$q = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1' - \sigma_2')^2 + (\sigma_2' - \sigma_3')^2 + (\sigma_1' - \sigma_3')^2}$$

Normal Consolidation Line and Unloading and Reloading Curves
Normally consolidated line

\[ e = e_N - \lambda \ln p' \]

Unloading - reloading line

\[ e = e_C - \kappa \ln p' \]

- Any point on the normally consolidated line represents the void ratio and state of stress for a normally consolidated soil.
- Any point on the unloading-reloading line represents and overconsolidated state.
- The material parameters \( \lambda, \kappa \) and \( e_N \) are unique for a particular soil.
  - \( e_N \) is the void ratio on the normally consolidated line that corresponds to 1 unit stress (i.e., 1 kPa). However, \( e_N \) may vary if other stress units are used.
- The slope of the critical state line parallels the normally consolidated line and both have a slope of \( \lambda \).
- The void ratio of the critical state line at \( p' = 1 \) kPa (or other unit pressure) is:
  \[ e_\Gamma = e_N - (\lambda - \kappa) \ln 2 \]
The critical state line is obtained by performing CD triaxial tests.

The slope of the critical state line, $M$, is related to the critical state friction angle by:

$$M = \frac{6 \sin \phi'}{3 - \sin \phi'}$$

The shear stress at the critical state can be found from:

$$q_f = Mp'_f$$

The void ratio at failure (i.e., critical state) is found by:

$$e_f = e_\Gamma - \lambda \ln p'$$

The yield curve for the MCC model is an ellipse in $p'$-q space:

$$q^2/p''^2 + M(1-p'_c/p') = 0$$
Strain hardening behavior for lightly overconsolidated clay

The effective stress path for a CD is a 3:1 slope (see text pp. 29-30).

Stress-Strain Curve showing strain hardening

Note that in the non-linear range of this stress-strain curve, the shear resistance is slightly increasing. This represents strain hardening.
Strain softening behavior for heavily overconsolidated clay

Note the yield surface decreases until the returning stress path touches the critical state line.

Stress-Strain Curve showing strain softening

Note that in the non-linear range of this stress-strain curve, the shear resistance is decreasing. This represents strain softening.
Stress dependency of bulk modulus in MCC model

- $K = (1 + e_0)p'/\kappa$

Other elastic parameters expressed in terms of stress dependency

- $E = 3 (1-2\nu)(1 + e_0)p'/\kappa$
- $G = 3 (1-2\nu)(1 + e_0)p'/\kappa / (2(1+\nu)\kappa)$
Calculating incremental plastic strains

- Once the yield surface is reached, a part of the strain is plastic (i.e., irrecoverable). The incremental total strain (elastic and plastic parts) can be calculated from:
  - Volumetric strain
    \[ d\varepsilon_v = d\varepsilon_v^e + d\varepsilon_v^p \]
  - Shear strain
    \[ d\varepsilon_s = d\varepsilon_s^e + d\varepsilon_s^p \]

For the triaxial state of stress

- \( d\varepsilon_v^p = d\varepsilon_1^p + 2d\varepsilon_3^p \)
- \( d\varepsilon_s^p = \frac{2}{3}(d\varepsilon_1^p - d\varepsilon_3^p) \)

Roscoe and Burland (1968) derived an associated plastic flow rule which describes the ratio between incremental plastic volumetric strain and incremental plastic shear strain. It is:

- \( \frac{d\varepsilon_v^p}{d\varepsilon_s^p} = \frac{(M^2 - \eta^2)}{2\eta} \)

where \( \eta = q/p' \) and at failure \( \eta = M \)
Determination of plastic strain increment

Note the normality rule states that the incremental volumetric and shear strains are perpendicular to each other.
Calculation of incremental volumetric and shear strains (plastic and elastic parts = total strain)

**Volumetric strains**  The plastic volumetric strain increment

\[ d\varepsilon^p_v = \frac{\lambda - \kappa}{1 + e} \left( \frac{dp'}{p'} + \frac{2\eta d\eta}{M^2 + \eta^2} \right) \]

The elastic volumetric strain increment

\[ d\varepsilon^e_v = \frac{\kappa}{1 + e} \frac{dp'}{p'} \]

Thus, the total volumetric strain increment:

\[ d\varepsilon_v = \frac{\lambda}{1 + e} \left[ \frac{dp'}{p'} + \left( 1 - \frac{\kappa}{\lambda} \right) \frac{2\eta d\eta}{M^2 + \eta^2} \right] \]

**Shear strains**

\[ d\varepsilon_s = d\varepsilon_s^p = \frac{\lambda - \kappa}{1 + e} \left( \frac{dp'}{p'} + \frac{2\eta d\eta}{M^2 + \eta^2} \right) \frac{2\eta}{M^2 - \eta^2} \]

or

\[ d\varepsilon_s = d\varepsilon_s^p = d\varepsilon_v^p \frac{2\eta}{M^2 - \eta^2} \]
Consolidated Drained Test Behavior of Lightly Overconsolidated Clay
Consolidated Undrained Test Behavior of Lightly Overconsolidated Clay
FLAC implementation

- incremental hardening/softening elastoplastic model
- nonlinear elasticity and a hardening/softening behavior governed by volumetric plastic strain ("density" driven)
- failure envelopes are similar in shape and correspond to ellipsoids of rotation about the mean stress axis in the principal stress space
- associated shear flow rule
- no resistance to tensile mean stress
Generalized Stress Components in Terms of Principal Stresses

\[ p = -\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \]
\[ q = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \]  

(2.178)

Volumetric and Distortional (i.e., shear) Strain Increments

The incremental strain variables associated with \(-p\) and \(q\) are the volumetric strain increment \(\Delta e\) and distortional strain increment \(\Delta e_q\), and we have:

Volumetric strain increment

\[ \Delta e = \Delta e_1 + \Delta e_2 + \Delta e_3 \]
\[ \Delta e_q = \frac{\sqrt{2}}{3} \sqrt{(\Delta e_1 - \Delta e_2)^2 + (\Delta e_2 - \Delta e_3)^2 + (\Delta e_1 - \Delta e_3)^2} \]  

(2.179)

Distortional strain increment

where \(\Delta e_j, j = 1, 3\) are principal strain increments. By assumption, the principal strain increments may be decomposed into elastic and plastic parts so that

Principal volumetric strain increments have an elastic and plastic part

\[ \Delta e_i = \Delta e_i^e + \Delta e_i^p \quad i = 1, 3 \]

Elastic part  Plastic part

(Note: \(e\) is volumetric strain and not void ratio)
The specific volume $v$ is defined as

$$v = \frac{V}{V_s} \tag{2.181}$$

where $V_s$ is the volume of solid particles, assumed incompressible, contained in a volume, $V$, of soil. The incremental relation between volumetric strain, $\varepsilon$, and specific volume has the form

$$\Delta \varepsilon = \frac{\Delta v}{v} \tag{2.182}$$

Starting with an initial specific volume $v_0$, we may thus write, for small volumetric strain increments,

$$v = v_0(1 + \varepsilon) \tag{2.183}$$

where $\varepsilon$ is the current accumulated volumetric strain.

The incremental expression of Hooke’s law in principal axes may be expressed in the form:

$$\Delta \sigma_1 = \alpha_1 \Delta e_1^\varepsilon + \alpha_2 (\Delta e_2^\varepsilon + \Delta e_3^\varepsilon)$$
$$\Delta \sigma_2 = \alpha_1 \Delta e_2^\varepsilon + \alpha_2 (\Delta e_1^\varepsilon + \Delta e_3^\varepsilon)$$
$$\Delta \sigma_3 = \alpha_1 \Delta e_3^\varepsilon + \alpha_2 (\Delta e_1^\varepsilon + \Delta e_2^\varepsilon) \tag{2.184}$$

where $\alpha_1 = K + 4G/3$; and

$$\alpha_2 = K - 2G/3.$$ 

In the Cam-clay model, the tangential bulk modulus $K$ in the volumetric relation above is updated to reflect a nonlinear law derived experimentally from isotropic compression tests. The results of a typical isotropic compression test are presented in the semi-logarithmic plot (next page).
As the normal consolidation pressure, $p_1$, increases, the specific volume, $v$, of the material decreases. The point representing the state of the material moves along the normal consolidation line defined by the equation

$$v = v_\lambda - \lambda \ln \frac{p}{p_1}$$  \hspace{1cm} (2.186)

where $\lambda$ and $v_\lambda$ are two material parameters, and $p_1$ is a reference pressure. (Note that $v_\lambda$ is the value of the specific volume at the reference pressure.)

**Note that the term "swelling" used above could be replaced with "reloading."**

An unloading-reloading excursion, from point $A$ or $B$ on the figure, will move the point along an elastic swelling line of slope $\kappa$, back to the normal consolidation line where the path will resume. The equation of the swelling lines has the form

$$v = v_\kappa - \kappa \ln \frac{p}{p_1}$$  \hspace{1cm} (2.187)
Elastic (i.e., recoverable) change in specific volume

\[ \Delta v^e = -\kappa \frac{\Delta p}{p} \]

After dividing by sides by the specific volume produces the relation between elastic changes in specific volume and changes in pressure

\[ -\Delta p = \frac{v_p}{\kappa} \Delta e^e \]

(The negative sign is needed because increases in pressure cause a decrease in specific volume.)

The tangential bulk modulus can be written as:

\[ K = \frac{v_p}{\kappa} \]

(Note this is different than an elastic bulk modulus because \( K \) is nonlinear function of \( p \) (mean effective stress).)
General Loading Conditions with Yielding

![Diagram showing plastic volume change](image)

**Figure 2.25** Plastic volume change corresponding to an incremental consolidation pressure change

\[ \Delta v^p = - (\lambda - \kappa) \frac{\Delta p_c}{p_c} \]  

Elastic (recoverable) change in specific volume.

\[ \Delta e^p = \frac{\lambda - \kappa}{v} \frac{\Delta p_c}{p_c} \]  

Plastic principal volumetric strain increment
Yield Function

\[ f = q^2 + M^2 p(p - p_c) \]

yield condition \( f = 0 \) is represented by an ellipse
horizontal axis \( p_c \) and vertical axis \( Mp_c \) in the \((q, p)\) plane

Associated flow rule

\[ g = q^2 + M^2 p(p - p_c) \]
Properties required for MCC model in FLAC

- **bulk_mod**: maximum elastic bulk modulus, $K_{max}$
- **density**: mass density, $\rho$
- **kappa**: slope of elastic swelling line, $\kappa$
- **lambda**: slope of normal consolidation line, $\lambda$
- **mm**: frictional constant, $M$
- **mpc**: preconsolidation pressure, $p_c$
- **mv0**: initial specific volume, $v_0$ (calculated internally, by default)
- **mp1**: reference pressure, $p_1$
- **mv_l**: specific volume at reference pressure, $p_1$, on normal consolidation line, $v_\lambda$
- **poiss**: Poisson's ratio, $\nu$
- **shear_mod**: elastic shear modulus, $G$

**Note use mv_l not mv_1**
\[ M = \frac{6 \sin \phi'}{3 - \sin \phi'} \quad \text{For triaxial compression} \]

\[ M = \frac{6 \sin \phi'}{3 + \sin \phi'} \quad \text{For triaxial extension} \]

\[ \lambda = \frac{C_s}{\ln(10)} \quad \text{From isotropic compression} \]

\[ \kappa \approx \frac{C_s}{\ln(10)} \quad \text{Usually 1/5 to 1/3 of } \lambda \]

\[ v_0 = v_\lambda - \lambda \ln \left( \frac{p_{c0}}{p_1} \right) + \kappa \ln \left( \frac{p_{c0}}{p_0} \right) \]

**Initial specific volume**

(Calculated by FLAC)

(see below)

\[ V_\lambda = \frac{V}{Vs} \quad \text{Specific Volume at } p_1 \]

\( V = \text{total volume} \)

\( Vs = \text{volume of solids} \)

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**Figure 2.27 Determination of initial specific volume**

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Notes on Poisson's ratio

If Poisson’s ratio, poiss, is not given, and a nonzero shear modulus, shearmod, is specified, then the shear modulus remains constant: Poisson’s ratio will change as bulk modulus changes. If a nonzero poiss is given, then the shear modulus will change as the bulk modulus changes: Poisson’s ratio remains constant. (The latter case usually applies to most problems.)

Properties for plotting

The following calculated properties can be printed, plotted or accessed via FISH.

12. \textit{bulk\textunderscore current}  current elastic bulk modulus, \(K\)
13. \textit{cam\textunderscore p}  effective pressure, \(p\)
14. \textit{cam\textunderscore q}  shear stress, \(q\)
15. \textit{ev\textunderscore plastic}  accumulated plastic volumetric strain
16. \textit{ev\textunderscore tot}  accumulated total volumetric strain
17. \textit{sv}  current specific volume

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More Reading

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11:43 AM

- Applied Soil Mechanics with ABAQUS Applications, pp. 28-53
- Applied Soil Mechanics with ABAQUS Applications, Ch. 5
- FLAC User's Manual, Theory and Background, Section 2.4.7
1. Modify the FISH code given below to model an axisymmetrical strain-controlled unconfined compression test on an EPS cylinder with a height of 5 cm and a diameter of 2.5 cm using a 5 x 20 uniform grid. The EPS should be modeled using the M-C using a density of 20 kg/m$^3$, Young's modulus of 5 MPa, Poisson's ratio of 0.1 and a cohesion of 50 KPa. You should include: a) plot of the undeformed model with boundary conditions, b) plot of the deformed model at approximately 3 percent axial strain, c) plot of axial stress vs axial strain, d) calculation of Young's modulus and unconfined compressive strength from c). (Note that the axial strain should be calculated along the centerline of the specimen.) (20 points).

```fish
config
set = large; large strain mode
grid 18,18; for 18'' x 18'' EPS block
model mohr
prop density = 20 bulk = 2.08e6 shear = 2.27e6 cohesion=50e3 friction=0 dilation=0 tension = 100e3;
EPS
ini x mul 0.0254; makes x grid dimension equal to 0.0254 m or 1 inch
ini y mul 0.0254; makes y grid dimension equal to 0.0254 m or 1 inch
fix y j 1; fixes base
;fix y i 8 12 j 1 ; fixes only part of base
his unbal 999
apply yvelocity -5.0e-6 from 1,19 to 19,19 ;applies constant downward velocity to simulate a strain-controlled test
def verticalstrain; subroutine to calculate vertical strain
whilestepping
avgstress = 0
avgstrain = 0
loop i (1,izones)
loop j (1,jzones)
vstrain = ((0 - ydisp(i,j+1) - (0 - ydisp(i,j)))/0.0254)*100 ; percent strain
vstress = syy(i,j)*(-1)
avgstrain = avgstrain + vstrain/18/18
avgstress = avgstress + vstress/18/18
end_loop
end_loop
end
his avgstrain 998
his avgstress 997
history 999 unbalanced
cycle 3000
```

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2. Modify the FLAC FISH code developed in problem 1 to simulate an axisymmetrical strain-controlled, consolidated drained triaxial compression test on a cylinder of sand that has a height of 5 cm and a diameter of 2.5 cm using a 5 x 20 uniform grid. The sand should be modeled using the M-C using a density of 2000 kg/m$^3$, Young's modulus of 10 MPa, Poisson's ratio of 0.3 and a drained friction angle of 35 degrees. The sample is first consolidated using a confining stress of 50 kPa and then sheared to failure. Your solution should include: a) plot of the undeformed model with boundary conditions, b) plot of the deformed model at approximately 3 percent axial strain, c) plot of axial stress vs axial strain d) plot of $p'$ vs. $q'$ e) calculation of Young's modulus and drained friction angle from c) and d) (20 points).

3. Change the constitutive relationship in problem 2 to a Modified Cam Clay model where $\lambda$ is 0.15, $\kappa$ is 0.03, $\nu$ is 0.3 (remains constant) and the initial void ratio of the same is 1.0 at 1 kPa. Model the same test as described in in problem 2 and provide the same required output. In addition, develop a comparative plot of plot of axial stress vs axial strain for the MC and MCC model results (20 points).