Interpretation of Effective Stress Friction Angle from In-Situ Tests

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Effective Stress Friction Angle, $\phi'$
- Drained frictional response of soils (use secant $\phi'$ and $c' = 0$)
- Effective frictional envelope for dry sands or saturated geomaterials (undrained, drained, static, cyclic)
- Drained penetration in uncemented clean quartz sands (SPT, CPT, DMT, PMT)
- Undrained penetration in all soil types by piezocone (CPTu) - NTH method

Friction Angle of Monterey Sand - Marachi et al. (1981)

Critical State Framework for $\phi'$ of Sands
(Bolton, March 1986, Geotechnique)
- Baseline friction angle for sand mineralogy and shape, $\phi_{cv} = \phi_{cs}$ (< 33° quartzitic, 40° feldspathic)
- Peak $\phi'$ above this essentially dilatancy effect where $\psi = $ dilatancy angle:
  - $\phi' (PSC) = \phi_{cv} + 5 \, I_{RD}$
  - $\phi' (TC) = \phi_{cv} + 3 \, I_{RD}$
- where $I_{RD} = D_0 \left[ Q - \ln(100 \, p_f'/\sigma_{atm}) - 1 \right] = $ the relative dilatancy index. Use $p_f' \approx 2 \sigma_{vo}'$
- Note: $Q = 10$ for quartz & feldspar; $= 8$ limestone, $5.5$ for chalk.
Relative Density (DR) of Clean Unaged Quartzitic Sands from SPT and CPT

Relative Density, DR (%)

<table>
<thead>
<tr>
<th>Normalized Resistance, (N₁)₆₀</th>
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<tbody>
<tr>
<td>0</td>
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<table>
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<tr>
<th>Overburden Pressure - kgf/cm²</th>
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<tbody>
<tr>
<td>0</td>
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<td>0</td>
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<tr>
<th>Normalized Tip Stress, qᵯ</th>
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<tbody>
<tr>
<td>0</td>
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<td>0</td>
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Reid-Bedford*
Platte River*
Standard Concrete*
Coarse (Gibbs & Holtz '57)
Fine (Gibbs & Holtz '57)
Field Sites (Skempton '86)
Terzaghi & Peck '48
Marcuson & Bieganowsky '77

OCR = (qD T)(100 - ND R)

Friction Angle of Sands from SPT - Schmertmann (1975)

Friction Angle of Sands from CPT from Russian Experience (Trofimenkov, 1974)

Friction Angle of Sands from CPT from Bearing Capacity Theory (Robertson & Campanella, 1983)

Effective φ' of Sands from SPT - N Value (Hatanaka & Uchida, Soils & Foundations, 1996)

\[ \phi' = [15.4(N₁)₆₀^{0.5} + 20]° \]

Triaxial Database from Frozen Sand Samples

Friction Angle, φ' (deg)

Normalized (N₁)₆₀

Sand (SP and SP-SM)
Sand Fill (SP to SM)
SM (Field trial)
H&T (1996)

φ' = \arctan[0.1 + 0.38 \log (qᵯ/σᵥ')]

Chamber Tests Uncorrected for Limited Boundary Effects

qc proportional with \sqrt{σᵥ'}

CPT from Bearing Capacity Theory (Robertson & Campanella, 1983)
**Friction Angle of Sands from CPT (Robertson & Campanella, 1983)**

Based Partially on CPT Calibration Chamber Tests
Uncorrected for Boundary Effects

$q_c$ directly proportional to $\sigma'_{vo}$

Evaluate $\phi'$ from CPT in clean quartz sands

- Assume: $\phi'(\text{peak}) \propto q_c/(\sigma'_{vo})^{0.5}$
  - Trofimenkov (1974) from Russian experience
  - Chamber test data (Kulhawy & Mayne 1990)
  - $\phi'(\text{deg}) \approx 17.6 + 11 \log \left[ q_c/(\sigma'_{vo})^{0.5} \right]$
- Assume: $\phi'(\text{peak}) \propto q_c/\sigma'_{vo}$
  - Bearing Capacity Theory - Limit Plasticity
  - Cavity Expansion Theory
  - Robertson & Campanella (1983) method
  - $\phi'(\text{deg}) = \arctan \left[ 0.1 + 0.38 \log (q_c/\sigma'_{vo}) \right]$

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**Friction Angle of Sands from CPT**

CPT Calibration Chamber Database Corrected for Boundary Effects (Kulhawy & Mayne, 1990)

$\phi'(\text{deg}) = 17.6 \circ + 11 \log \left[ q_t/\sigma'_{vo}^{0.5} \right]$

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**GT Test Site, West Campus**

Atlanta, Georgia

Drilled Shaft Load Test Program

Sponsored by:
- Association of Drilled Shaft Contractors (ADSC)
- Federal Highway Administration (FHWA)
- American Society of Civil Engineers (ASCE) Georgia Section

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**GT Test Site, West Campus**

- Association of Drilled Shaft Contractors
- Federal Highway Administration
- ASCE Atlanta Geotechnical Chapter
Effective Stress, $p' = (\sigma_1' + \sigma_3')/2$ (kPa)

Shear, $q = (\sigma_1' - \sigma_3')/2$ (kPa)

$\phi' = 36.0^\circ$

$c' = 0$

Triaxial Summary – GT Campus

SPT Summary - Georgia Tech

Effective Friction Angle, $\phi'$ (deg)

Correlation by Hatanaka & Uchida, 1996

Triaxial

SPT Summary - Georgia Tech

Effective Friction Angle, $\phi'$ (deg)

H & U'96

Triaxial

SPT Summary - Georgia Tech

Effective Friction Angle, $\phi'$ (deg)

H & U'96

Triaxial

CPT $q_t$ (C&R'83)

Triaxial

CPT $q_t$ (K&M'90)

Triaxial Tests

ADSC Load Test at West GT Campus

Effective Stress Friction Angle, $\phi'$ (deg)

CPT $q_t$ (C&R'83)

Triaxial

CPT $q_t$ (K&M'90)

Triaxial Tests
CIDC Triaxial Results on Frozen Sand Samples (Mimura, 2003)

\[ \phi' = 17.6 + 11.0 \cdot \log \left( \frac{q_t}{\sigma'_{vo}} \right) \]

Yodo River
Natori River
Tone River
Edo River
K&M90

GT West Campus Test Site

DMTs at GT West Campus Test Site

Effective Friction Angle \( \phi' \) (deg)

Marchetti, 1997
Marchetti, 1996
C & R 1991
Lab Triaxial

GT West Campus Test Site

Effective Stress Piezocone Penetration
Senneset, Sandven, and Janbu (TRR 1989)

Sands, Silts, and Clays

Senneset, Sandven, and Janbu (TRR 1989)

Effective Stress Strength Parameters

- **Bearing Capacity Theories (sands)**
  - Durgunoglu & Mitchell (1975); Vesic (1977); Robertson & Campanella (1983);
  - Salgado et al. (1994); Jamiolkowski & LoPresti (2000)

- **CSSM Dilatancy Approach using \( D_B \) from CPT in Sands** (Bolton, 1986)

- **Effective Stress Method for all soil types**
  - Senneset, Janbu & Sandven, 1989
Effective Stress Strength Parameters

- Effective Stress Method for all soil types (Senneset, Janbu & Sandven, 1989)

“Sci-Fi”

Professor Mike Jamiolkowski
Politecnico Torino

NTH Effective Penetration Theory
Ref: Senneset, et al. (1982, ESOPT)

- Cone Resistance Number ($N_m$)
  \[ N_m = \frac{1}{1 + N_u} = \frac{q_t - \sigma_{vo}}{\sigma_{vo}' + \alpha} \]

- Intercept = attraction $a' = c' \cot \phi'$

- Porewater Bearing Factor, $N_u$
  \[ N_u = 6 \cdot \tan \phi' (1 + \tan \phi') \]

- End Bearing Factor, $N_q$
  \[ N_q = \tan^{-1}(45^\circ + \frac{\pi}{2} \phi') \exp\left(\pi - 2\beta \tan \phi'\right) \]

Drained Penetration: $q_t + a' = N_q (\sigma_{vo}' + \alpha')$

NTH Effective Penetration Theory

- Cone Resistance Number ($N_m$) = measured slope of ($q_t - \sigma_{vo}$) vs. $\sigma_{vo}'$

- Intercept = attraction $a' = c' \cot \phi'$

- If $a' = 0$ assumed: $N_m = Q = (q_t - \sigma_{vo})/\sigma_{vo}'$

- Porewater Parameter: $B_q = \Delta u_2/(q_t - \sigma_{vo})$

- Use solution for $\beta = 0$ (angle of plastification)

- $Q$ and $B_q$ same as in Robertson (1990) soil behavioral classification charts.

Gloucester Test Site, Ontario
(Konrad & Law, Canadian Geot. J., August 1987)

Piezocone Readings (kPa)

Depth (meters)

Canadian National Test Site, Gloucester, Ontario

Gloucester CPTu, Ontario

Net Cone Stress, $q_t - \sigma_{vo}$ (kPa)

Drain Gage, $u_2$ (kPa)

$N_m = 6.3$

$B_q = 0.65$

Robertson & Campanella (1983)

(Senneset et al., ISOPT-1988, TRR 1989)
**Gloucester, Ontario**

Senneset & Janbu (1985)

Effective Stress Penetration

- Forene $\phi' = 34.5^0$


**Approximation of NTH-\(\phi'\) for Bq > 0.1**

- \(N_m = Q\)
  \[N_m = \frac{(q_t - \sigma_v)}{\sigma_v'}\]

- \(\phi' = 34.5^0\)

**Gloucester Test Site, Ontario**

**Approx. NTH Method - Gloucester, Ontario**

**SCPTu Sounding, Sandpoint, Idaho**

**Evaluating \(\phi'\) at Sandpoint, Idaho (NTH Method)**

- \(N_m = 3.43\)
Evaluation of $\phi'$ at Sandpoint, Idaho (NTH Method)

**Effective Stress Penetration (NTH Method)**

- $B_q = 0.80$

- Parameters: $\Delta u_2$ (kPa) vs. Net Cone Tip Resistance, $q_t-\sigma_{vo}$ (kPa)

Evaluation of $\phi'$ by CPTu (NTH Method)

- $B_q = 0.1$

- $\phi' = 32.3^\circ$

Interpretation of Effective Friction Angle

**Maximum q Criterion - TRIAXIAL DATA SUMMARY - IDAHO**

- $\phi' = 32.3^\circ$
- $c' = 0$ kPa

CPTu Evaluations in Silty Clays, Sandpoint, Idaho

- Normalized $q_t$
- Normalized $B_q$
- Effective $\phi'$