Freeway Traffic State Estimation and Uncertainty Quantification based on Heterogeneous Data Sources: Stochastic Three-Detector Approach

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Abstract

This study focuses on how to use multiple data sources, including loop detector counts, AVI Bluetooth travel time readings and GPS location samples, to estimate microscopic traffic states on a homogeneous freeway segment. A multinomial probit model and an innovative use of Clark’s approximation method were introduced to extend Newell’s method to solve a stochastic three-detector problem. The mean and variance-covariance estimates of cumulative vehicle counts on both ends of a traffic segment were used as probabilistic inputs for the estimation of cell-based flow and density inside the space-time boundary and the construction of a series of linear measurement equations within a Kalman filtering estimation framework. We present an information-theoretic approach to quantify the value of heterogeneous traffic measurements for specific fixed sensor location plans and market penetration rates of Bluetooth or GPS floating car data.

Key words: kinematic wave method, multinomial probit model, Clark’s approximation, traffic state estimation

1. Introduction

By reducing traffic system instability and volatility, the transportation system will operate more efficiently, with better end-to-end trip travel time reliability and reduced total emissions. By closely monitoring and reliably estimating the state of the system using heterogeneous data sources, it is possible to apply information provision and control actions in real time to best utilize the available highway capacity. These two realizations have motivated the two main directions of this research: estimating freeway traffic states from heterogeneous measurements and quantifying the uncertainty of traffic state estimations under different sensor network deployment plans.

1.1. Literature review

A majority of modeling methods focus on macroscopic point bottleneck detection and link-level travel time estimation problems (e.g., Ashok and Ben-Akiva, 2000; Zhou and List, 2010; Coifman, 2002). Recently, a number of data-mining methods have been proposed for the purpose of obtaining microscopic traffic states on freeway segments using different sources of data.

A generic microscopic traffic state estimation method consists of a number of key components: an underlying traffic flow model, a state variable representation, and a system process and a measurement equation. Different traffic flow models could lead to various system state representation and process equations. For example, the Cell Transmission Model (CTM), proposed by Daganzo (1994), captures the transfer flow volume between cells as a minimum of sending and receiving flows, while Newell’s simplified kinematic wave model (Newell, 1993), or three-detector method, which has been systematically described by Daganzo (1997), considers cumulative vehicle counts at an intermediate location of a homogeneous freeway segment as a minimization function of the upstream and downstream cumulative arrival and departure counts.

To apply computationally efficient filters (e.g., a Kalman filter or particle filter) to handle large-volume streaming sensor data, one of the major modeling challenges for traffic state estimation is how to extract or construct linear system processes and measurement equations. The widely used Eulerian sensing framework (e.g., Muñoz et al., 2003; Sun et al., 2003; Sumalee et al., 2011) uses linear measurement equations to incorporate flow and speed data from point detectors, while the emerging Lagrangian sensing framework (e.g., Nanthawichit et al., 2003; Work et al., 2010; Herrera and Bayen, 2010) aims to establish linear measurement equations to utilize semi-continuous samples from moving observers or probes.
Muñoz et al. (2003) proposed a novel switching-mode model (SMM), which adapts a Modified Cell Transmission Model (MCTM) to describe traffic dynamics and transforms its nonlinear (minimization) state equations into a set of piecewise linear equations. In particular, each set of linear equations is referred to as a mode, and the SMM switches between different modes according to the detailed congestion status of the cells in a section and the values of the mainline boundary inputs. Along this line, Sun et al. (2003) employed a mixture Kalman filter to approximate the probabilistic state space through a finite number of mode sample sequences, where the weight of each sample is dynamically adjusted to reflect the posterior probability of all state vectors. Sumalee et al. (2011) further introduced stochastic elements to the MCTM framework by Muñoz et al. (2003) and proposed a stochastic cell transmission model.

Based on a second-order traffic flow model, Wang and Papageorgiou (2005) and Wang et al. (2007) presented a comprehensive extended Kalman filter framework for the estimation and prediction of highway traffic states. To construct linear process equations, linearization around the current state (typically segment density) is required to determine the outgoing flows between segments. Mihaylova et al. (2007) developed a CTM-based second-order macroscopic model and adopted an alternative particle-filtering framework to avoid computationally intensive linearization operations.

Nanthawichit et al. (2003) conducted an early study that used Payne’s traffic flow model and Kalman filtering within a Lagrangian sensing framework. Work et al. (2010) derived a velocity-based partial differential equation (PDE) to construct linear measurement equations for utilizing Lagrangian data, while an Ensemble Kalman filter was embedded to propagate non-linear state equations through a Monte Carlo simulation approach. Herrera and Bayen (2010) incorporated a correction term to the Lighthill-Whitham-Richards partial differential equation (Lighthill and Whitham, 1955; Richards, 1956) to reduce the discrepancy between the Lagrangian measurements and the estimated states. Treiber and Helbing (2002) proposed an efficient interpolation method by first employing a “kernel function” to build the state equation for forward and backward waves, and then integrating these two equations into a linear state equation through a speed measurement-based weighting scheme. Based on the cumulative flow count and simplified kinematic wave model (Newell, 2003), Coifman (2002) developed methods to reconstruct vehicle trajectories from the measured local speed measures or a partial set of vehicle probe trajectories. While Mehran et al. (2011) further investigated the sensitivity impact of input data uncertainty, their solution framework has not directly taken into account the measurement errors of different data sources.

1.2. Overview

While significant progress has been made in formulating system process and measurement equations for the freeway traffic state estimation problem, this study aims to address several challenging theoretical and practical issues.

First, we propose a stochastic version of Newell’s three-detector model to utilize multiple data sources to estimate microscopic traffic states for a homogeneous freeway segment. This method provides a new alternative to the existing CTM-based traffic state estimation approach and the interpolation method of Treiber and Helbing (2002). In particular, the traffic state of any intermediate point on a freeway segment can be estimated directly from the boundary conditions through a minimization operation. To handle the upstream and downstream cumulative flow counts as two random variables, we introduce a multinomial probit model and Clark’s approximation (from the field of discrete choice modeling) to approximate the minimization of two random variables as a third random variable with quantifiable mean and variance. By doing so, we could link the accuracy of traffic state estimation for each cell directly with the variability of the boundary conditions.

Second, this study aims to incorporate emerging Automatic Vehicle Identification (AVI) and Global Positioning System (GPS) data to estimate the inside microscopic states of a traffic segment. There are a number of surveillance techniques available for the purposes of traffic monitoring and management. Each technique has the ability to collect and process specific types of real-time traffic data. AVI data, which are obtainable from mobile phone Bluetooth samples, represent an emerging data source, but they have been mainly used in link-based travel time estimation applications (e.g., Wasson et al., 2008, Haghani et al., 2010) or origin-destination demand estimation (e.g., Zhou and Mahmassani, 2006) rather than in the estimation of within-link traffic states, such as cell-based density. The existing Lagrangian sensing framework (Nanthawichit et al., 2003; Work et al., 2010; Herrera and Bayen, 2010) can map location-based speed samples to a moving observer-oriented PDE system, but it has difficulties in incorporating end-to-end time-dependent travel time records from AVI readers across a series of cells.

It is practically important but theoretically challenging to utilize AVI data. In our proposed approach, both AVI and GPS samples can be viewed as “bridges” between the upstream and downstream boundaries in terms of cumulative flow counts. Specifically, we develop a series of linear measurement equations within the proposed
stochastic three-detector approach that can dramatically simplify the process of estimating the likelihood of free-flow vs. congested traffic conditions for any location inside a traffic segment. Third, the value of information (VOI) for the highway traffic state estimation problem systematically investigated for various types of data sources. We use an information-theoretic approach to quantify the uncertainty of microscopic traffic state estimation results and further evaluate the effectiveness of various important sensor design scenarios, such as point detector sampling rates, AVI market penetration rates, and GPS market penetration rates.

Table 1 summarizes the data measurement types and comparative advantages of estimating traffic states at different resolutions. Each of these data sources has strengths and weaknesses, and an effective traffic state monitoring system must be able to fuse multiple data streams to symmetrically capture traffic system instability and volatility. Moreover, as more sensing technologies become available, the monitoring system must be able to seamlessly incorporate them into a computationally efficient and theoretically rigorous analysis framework.

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This paper is organized as follows. After describing the highway traffic state estimation problem, Section 2 briefly reviews the deterministic three-detector model, which is based on the triangular relationship and Newell’s method. In Sections 3, 4 and 5, we sequentially discuss stochastic boundary conditions and propose a generalized least squares estimation framework to solve the stochastic three-detector problem using heterogeneous data sources. In Section 6, numerical experiments are used to demonstrate the proposed methodology and illustrate observability improvements under different sensing plans and market penetration rates.

2. Problem statement and conceptual framework
2.1. Notation and input data

Parameters of traffic flow model

\( v_f \) = Free-flow speed in the free-flow state.
\( w_b \) = Backward wave speed in the congestion state.
\( k_j \) = Jam density or maximum density, where the flow reduces to zero.
\( \text{FFTT} \) = Time for traversing a certain distance by a forward wave with speed \( v_f \).
\( \text{BWTT} \) = Time for traversing a certain distance by a backward wave with speed \( -w_b \).

Subscripts and parameters of space-time representation

\( \Delta x \) = Unit space increment, i.e., length of one section.
\( i \) = Space index of sections, \( i = 1, \ldots, I \).
\( x \) = Space position of section \( i \), i.e., \( x = i\Delta x \).
\( x_u \) = Location of upstream boundary, \( x_u = 0 \).
\( x_d \) = Location of downstream boundary, \( x_d = I\Delta x \).
Consider a homogeneous freeway segment without enter or exit ramps in between. The segment is divided into a number of sections of $x = i \Delta x$, $i = 1, ..., J$, and $\Delta x$ is the length of a section. The modeling time horizon is discretized into $t = j \Delta t$, $j = 1, ..., J'$, where $j$ denotes the modeling time index, and $\Delta t$ denotes the length of each simulation time step. We use $t = j j'$, $j = 1, ..., J'$ to denote sampling time stamps, where $j'$ denotes the sampling time index, and $j'$ represents the length of each sampling time interval, e.g., 30 s or 5 min.
Two point sensor stations are located at upstream location \(x_u = 0\) and downstream location \(x_d = I\Delta x\). The measurement equation for the vehicle counts at the upstream sensor can be expressed as
\[
\tilde{c}_u(t) = c(t, x_u) + \ell_u(t),
\]
where \(\ell_u(t) \sim N(0, \delta_u)\).

Generally, the cumulative upstream vehicle counts at each sampling time stamp can be derived from the observed vehicle counts:
\[
\begin{equation}
\label{eq:3}
z_u(j\tau) = \sum_{t=0}^{j\tau} \tilde{c}_u(t) = \sum_{t=0}^{j\tau} c(t, x_u) + \sum_{t=0}^{j\tau} \ell_u(t),
\end{equation}
\]

where the summation \(\sum_{t=0}^{j\tau} \ell_u(t)\) of multiple normal random independent variables is the error of measured cumulative vehicle counts \(z_u(j\tau)\) at sampled time \(j\tau\). Because the sum of multiple normally distributed independent variables is normally distributed, the cumulative vehicle count \(z_u(j\tau)\) follows a normal distribution.

To construct the cumulative vehicle counts at the non-sampled time stamps, we employed a linear interpolation method, shown below. For a time stamp \(t \in [j\tau, (j+1)\tau]\), the corresponding cumulative vehicle counts can be derived as follows:
\[
\begin{equation}
\label{eq:4}
z_u(t) = z_u(j\tau) + \frac{z_u((j+1)\tau) - z_u(j\tau)}{\tau} \times (t - j\tau).
\end{equation}
\]

Assuming the upstream detector produces unbiased measurements, we can express the mean value of a continuous cumulative arrival flow count \(n_u(t)\) as
\[
\begin{equation}
\label{eq:5}
n_u(t) = \sum_{t=0}^{j\tau} c(t, x_u) + \frac{\sum_{t=0}^{(j+1)\tau} c(t, x_u) - \sum_{t=0}^{j\tau} c(t, x_u)}{\tau} \times (t - j\tau).
\end{equation}
\]

We can also derive the related error term \(E_u(t)\), which is the combined error source, including the measurement error \(\sum_{t=0}^{j\tau} \ell_u(t)\) and \(\sum_{t=0}^{(j+1)\tau} \ell_u(t) - \sum_{t=0}^{j\tau} \ell_u(t)\) and the linear interpolation error.

Likewise, the cumulative departure flow count curve at the downstream station can be constructed.

Given deterministic cumulative departure and arrival flow counts, \(n_u(t)\) and \(n_d(t)\), at the upstream and downstream detectors, the three-detector problem considered by Newell (1993) aims to determine the traffic state at any intermediate detector location \(x_u < x < x_d\). The traffic state at the third detector location \(x\) is represented by cumulative flow count value \(n(t, x)\), and cell-based density, flow and speed measures can be derived easily as functions of \(n(t, x)\).

![Fig. 1. Illustration to boundary condition: (a) deterministic boundary condition; (b) stochastic boundary condition.](attachment:image.png)
2.2. Newell's deterministic method for solving the three-detector problem

In Newell’s method for solving the deterministic three-detector problem, the cumulative vehicle counts \( n(t, x) \) of any point in the interior of the boundary can be directly evaluated from the boundary input \( n_u(t) \) and \( n_d(t) \). Recognizing two types of characteristic waves in the triangular shaped flow-density curve, the solution method includes a forward wave propagation procedure and a backward wave propagation procedure.

In the forward propagation procedure, a forward wave traverses free-flow travel time from upstream at time \( t = \frac{x}{v_f} \) to a generic point \( x \) at time \( t \). This leads to

\[
 n(t, x) = n_u \left( t - \frac{x}{v_f} \right). \tag{5}
\]

In the backward wave propagation procedure, a backward wave is emitted from the downstream boundary to the generic point \( x \) at time \( t \) inside the boundary. Because the wave pace of the backward wave is equal to \(-\frac{1}{w_b}\), and the density along the backward wave is \( k_j \) (according to the triangular shaped flow-density relationship), we have

\[
 n(t, x) = n_d \left( t - \frac{x_d - x}{w_b} \right) + k_j (x_d - x). \tag{6}
\]

Considering \( L_d \) as the distance from the downstream boundary to a point \( x \) inside the boundary, Newell’s method selects the smallest value of \( n(x, t) \) between estimated values from the forward and backward wave propagation procedure:

\[
 n(t, x) = \min \left( n_u \left( t - \frac{x}{v_f} \right), n_d \left( t - \frac{x_d - x}{w_b} \right) + k_j L_d \right). \tag{7}
\]

If either procedure leads to a flow \( \partial N(t, x)/\partial t \) that exceeded the capacity at \( x \), one needs to restrict \( n(t, x) \) by a straight line with a slope equal to the capacity at \( x \).

Hurdle and Son (2001, 2002) and Son (1996) demonstrated the effectiveness and tested the computational efficiency of Newell’s method using field data. Daganzo (2003, 2005) presented an extension to the variational formulation of kinematic waves, where the fundamental diagram is relaxed to a concave flow-density relationship. Furthermore, Daganzo (2006) showed the equivalence between the kinematic wave with a triangular fundamental diagram and a simplified linear car, following a model similar to the one proposed by Newell (2002).

2.3. Conceptual framework

Fig. 2 illustrates the conceptual framework of the proposed methodology. The conceptual framework starts from prior stochastic boundary estimates, which consists of a prior estimation of cumulative vehicle counts vector \( N^- \) in block 1 and a prior estimation of variance-covariance matrix \( P^- \) in block 2. These prior estimates of \( N^- \) and \( P^- \) can be extracted from historical information or available loop detector counts on both ends of a link. A series of linear measurement equations in block 3 are derived from the building blocks at the bottom half of Fig. 2. Specifically, we developed a generalized least squares estimation method (i.e., the updating step of the Kalman filter) to update the stochastic boundary in terms of the cumulative vehicle counts vector \( N^+ \) in block 4 and the posterior estimation variance-covariance matrix \( P^+ \) in block 5, which further provide the final estimates of cell-based flow and density in blocks 12 and 13. Based on detailed sensor network settings in block 6, we developed linear measurement equations from heterogeneous data sources in block 7, which was constructed from the multinomial probit model and Clark’s approximation in block 8 as well as Newell’s simplified kinematic wave model in block 9. This single set of linear measurement equations provides the key modeling elements of linear measurement matrix \( H \) in block 10 and measurement error variance and covariance matrix \( R \) in block 11.
3. Solving stochastic three-detector model using the multinomial probit model and Clark’s approximation

By extending Newell’s deterministic three-detector model as shown in Fig. 1(a), this section presents the model and solution algorithms for an STD problem, which aims to estimate the traffic state at any intermediate location $x_u < x < x_d$ on a homogeneous freeway segment using available measurements with various degrees of measurement errors. Mathematically, the proposed STD problem needs to consider a stochastic version of Eq. (7):

$$ n(t,x) = \min \left( z_u \left( t - \frac{x}{v_f} \right), z_d \left( t - \frac{L_p}{w_p} \right) + k_j L_D \right), $$

where both cumulative arrival and departure flow counts are Normal random variables, as shown previously,

$$ z_u \left( t - \frac{x}{v_f} \right) = n_u \left( t - \frac{x}{v_f} \right) + \mathcal{E}_u \left( t - \frac{x}{v_f} \right), \text{ and}$$

$$ z_d \left( t - \frac{L_p}{w_p} \right) = n_d \left( t - \frac{L_p}{w_p} \right) + \mathcal{E}_d \left( t - \frac{L_p}{w_p} \right). $$

The key to solving the proposed Eq. (8) is the development of efficient approximation methods to estimate the cumulative vehicle counts $n(t,x)$ at location $x$ at time $t$. By assuming that the maximum of two normally distributed random variables can be approximated by a third normally distributed random variable, Clark (1961) proposed an approximation method to calculate the mean and variance (i.e., the first two moments) of the third Normal variable. In the field of discrete choice modeling (Daganzo, 1979), a multinomial probit model has been widely used to calculate the choice probability of an alternative based on a utility-maximization or a disutility-minimization framework, where the unobserved terms of alternative utilities are assumed to be normal distributions with possible correlation and heteroscedasticity structures. Daganzo et al. (1977) and Horowitz et al. (1982) investigated the numerical accuracy of Clark’s approximation under a small number of alternatives.

By reformulating Eq. (8) within a disutility-minimization framework, the cumulative vehicle count $n(x,t)$ is the minimum of the above two disutilities, corresponding to the forward wave and backward wave alternatives.

$$ n(t,x) = \min(\mathcal{U}_u, \mathcal{U}_d), $$

Fig. 2. Conceptual framework of the proposed methodology
where \( U_u = V_u + \xi_u = z_u \left( t - \frac{x}{v_f} \right) \) and \( U_d = V_d + \xi_d = z_d \left( t - \frac{L_D}{w_p} \right) + k_d L_d. \) \((12)\)

It is easy to verify that the systematic disutility \( V_u = n_u \left( t - \frac{x}{v_f} \right) \) and \( V_d = n_d \left( t - \frac{L_D}{w_p} \right) + k_d L_D, \) respectively, correspond to the forward or backward wave propagation procedures in Eqs. (5-6). The unobserved terms can be derived as \( \xi_u = \mathcal{E}_u \left( t - \frac{x}{v_f} \right) \) and \( \xi_d = \mathcal{E}_d \left( t - \frac{L_D}{w_p} \right). \)

In this probit model framework, the choice probability of each alternative is equivalent to the probability of the forward wave vs. the backward wave being selected to determine the traffic state (i.e., free-flow vs. congested) of the current time-space location \((t, x)\). In this study, we further adopted Clark’s approximation method to estimate the mean and variance of the estimated cumulative flow count \( n(t, x) \) as

\[
n(t, x) \sim N \left( E_n(t, x), Var_n(t, x) \right), \tag{13}\]

where the mean
\[
E_n(t, x) = v_1 \tag{14}
\]

and the variance
\[
Var_n(t, x) = v_2 - v_1^2. \tag{15}\]

Based on the notation system used in Sheffi (1985), the coefficients \( v_1 \) and \( v_2 \) can be further calculated by the following formulas.

\[
v_1 = V_u \Phi(\gamma) + V_d \Phi(-\gamma) + \alpha \Phi(\gamma); \tag{16}\]
\[
v_2 = (V_u^2 + \sigma_u^2) \Phi(\gamma) + (V_d^2 + \sigma_d^2) \Phi(-\gamma) + (V_u + V_d)\alpha \Phi(\gamma); \tag{17}\]

There are several elements in Eqs. (16-17), including

(i) a parameter describing the standard deviation of the systematic disutility difference \( V_d - V_u \):

\[
\alpha = \sqrt{\sigma_u^2 + \sigma_d^2 - 2 \sigma_u \sigma_d \psi_{ud}}, \tag{18}\]

where \( \sigma_u \) and \( \sigma_d \) denote the variance of \( \xi_u \) and \( \xi_d \), respectively, and \( \psi_{ud} \) is the correlation coefficient between the error terms \( \xi_u \) and \( \xi_d \):

(ii) a standardized normal variable
\[
\gamma = \frac{V_d - V_u}{\alpha}, \tag{19}\]

(iii) a corresponding standard normal distribution function \( \Phi(\gamma) = (2\pi)^{-1/2} \exp \left( -\frac{\gamma^2}{2} \right) \) and a cumulative normal distribution curve

\[
\Phi(\gamma) = \int_{-\infty}^{\gamma} \phi(\tau) d\tau. \tag{20}\]

In particular, Eq. (16) also show that the relative weights for the systematic disutilities \( V_u \) and \( V_d \) in the final mean estimate \( E_n(x, t) \) are jointly determined by the cumulative distribution functions \( \Phi(\gamma) \) and \( \Phi(-\gamma) \) as well as an adjustment factor of \( \alpha \Phi(\gamma) \) that ranges between 0 and 1.

Because the deterministic three-detector model is a special case of the proposed STD model with error-free measurement, we can substitute \( \psi_{ud} = 0 \) and \( \psi_{ud} = 0 \) into Eqs. (14-20) to obtain the mean and variance of cumulative flow count \( n(x, t) \) in the following relationships between \( V_u \) and \( V_d \):

\[
E_n(t, x) = \begin{cases} 
V_u & \text{if } V_d > V_u, \text{then } \gamma = \infty, \quad \Phi(\gamma) = 1, \Phi(-\gamma) = 0 \\
V_d & \text{if } V_d = V_u, \text{then } \gamma = 1, \quad \Phi(\gamma) = 0.8413, \Phi(-\gamma) = 0.1587. \\
V_d & \text{if } V_d < V_u, \text{then } \gamma = -\infty, \quad \Phi(\gamma) = 0, \Phi(-\gamma) = 1
\end{cases} \tag{21}\]

\[
Var_n(t, x) = \begin{cases} 
0 & \text{if } V_d > V_u, \text{then } v_2 = V_u^2 \\
0 & \text{if } V_d = V_u, \text{then } v_2 = V_u^2 = V_d^2. \\
0 & \text{if } V_d < V_u, \text{then } v_2 = V_d^2
\end{cases} \tag{22}\]

When solving the deterministic three-detector model by Clark’s approximation method, we obtain an error-free cumulative vehicle count \( n(t, x) \) through the simple minimization operation. This derivation confirms that the proposed method using Clark’s approximation can satisfactorily handle the deterministic three-detector model as a special case of the STD model.
4. Measurement models for heterogeneous data sources

Corresponding to blocks 8 and 9 of the conceptual framework in Fig. 2, the previous session proposed approximation formulas that can connect internal state \( n(t, x) \) with the stochastic boundary conditions. This session proceeds to establish a set of linear measurement equations that can map additional sensor measurements to the boundary conditions \( n_u(t) \) and \( n_d(t) \). The following discussions detail the modeling components for blocks 3, 10 and 11 in Fig. 2 regarding the linear measurement equations shown below.

\[
Y = HN + \varepsilon, \text{ where } \varepsilon \sim N(0, R). \tag{23}
\]

Specifically, measurement vector \( Y \) can include flow counts and occupancy from additional point detectors, Bluetooth reader travel time measurements, and GPS vehicle trajectory data. Matrix \( H \) provides a linear map between cumulative vehicle counts on the boundary, namely \( n_u(t) \) and \( n_d(t) \), and observations \( Y \). The measurement error covariance matrix \( R \) is referred to as the combined error \( \varepsilon \) that includes error sources such as sensor measurement errors and approximation errors in the proposed modeling approach.

In general, more measurements would lead to less uncertainty in the boundary conditions. Fig. 3 illustrates three typical sensing configurations to reduce the estimation errors in the freeway traffic state estimation problem:

(i) deploying an additional point detector at the intermediate location, which can produce vehicle counts and occupancy measurements;

(ii) installing two prevailing AVI (e.g., mobile phone Bluetooth) readers, which can detect passing time stamps of individual vehicles;

(iii) equipping a certain percentage of vehicles with GPS mobile devices, which can produce semi-continuous vehicle trajectories for a short sampling interval, e.g., every 10 seconds.

![Fig. 3. Illustration of additional measurements from middle point sensor, AVI and GPS sensors.](image)

4.1. Measurement equations for vehicle counts and occupancy from additional point detectors

In the analysis time period \([0, T]\), an additional point sensor, located at \( x_m \), as shown in Fig. 3, produces \( T/\tau \) vehicle count measurements. For simplicity, let us first assume that the counting process starts from an empty segment at time \( t=0 \), and then we obtain a cumulative vehicle count \( z(j'\tau, x_m) \) at time stamp \( j'\tau \)

\[
z(j'\tau, x_m) = \sum_{\tau=0}^{j'\tau} c(t, x_m) + \sum_{\tau=0}^{j'\tau} \ell_m, \tag{24}
\]

where \( c(t, x_m) \) is the observed link volume covering time period \([t, t+\tau]\), \( \sum_{\tau=0}^{j'\tau} c(t, x_m) \) denotes the constructed cumulative flow counts, and \( \sum_{\tau=0}^{j'\tau} \ell_m \) denotes the measurement error term of \( z(j'\tau, x_m) \).

Within the proposed cumulative flow count-based estimation framework, the key to establishing a linear measurement equation is mapping vehicle count and occupancy measurements to the state value of \( n_u(t) \) and \( n_d(t) \). Through Clark’s approximation formula in Eqs. (13-19), we can map the constructed cumulative flow count \( \sum_{\tau=0}^{j'\tau} c(t, x_m) \) to the boundary conditions as
\[
\sum_{t=0}^{\tau} c(t, x_m) = |\Phi(\gamma) \Phi(-\gamma)| \times \begin{bmatrix}
n_u \left( t - \frac{x}{v_f} \right) \\
n_d \left( t - \frac{L_D}{w_b} + k_j L_D \right)
\end{bmatrix} + \alpha \Phi(\gamma) + \epsilon',
\]

where the combined error term \( \epsilon' \) includes both the measurement error \( \sum_{t=0}^{\tau} \ell_m(t) \) and the estimation error in Clark’s approximation, \( v_2 - v_1^2 \). Within the linear measurement framework
\[ Y' = H'N' + \epsilon', \]
where \( \epsilon' \sim N(0, R') \),
we can construct a transformed measurement of \( Y' = \sum_{t=0}^{\tau} c(t, x_m) - \Phi(-\gamma) \times k_j L_D - \alpha \Phi(\gamma) \), the mapping vector \( H' = [\Phi(\gamma) \Phi(-\gamma)] \), and the system state vector \( N' = \begin{bmatrix} n_u \left( t - \frac{x}{v_f} \right) \\
n_d \left( t - \frac{L_D}{w_b} \right) \end{bmatrix} \).

As an extension, if there are vehicles on the segment at time \( t=0 \), then we can reset \( n_d(t=0)=0 \) and adjust cumulative flow counts from the middle sensor to consider the additional number of vehicles that have already passed through \( x_m \) but have not reached the end of segment \( x_d \).

A dual loop detector that includes two detectors at location \( x_1 \) and \( x_2 = x_1 + l \), where \( l \) is the distance of the two detectors yields occupancy measurements that can be converted into local density \( \text{Cassidy and Coifman, 1997} \). By expressing the local density at time \( j' \tau \) at location \( \frac{x_1 + x_2}{2} \) as a function of the estimated cumulative vehicle count \( n(j' \tau, x_1) \) and \( n(j' \tau, x_2) \)
\[ k \left( j' \tau, \frac{x_1 + x_2}{2} \right) = n(j' \tau, x_1) - n(j' \tau, x_2), \]
we obtain the following linear measurement equations.
\[
1 \times k \left( j' \tau, \frac{x_1 + x_2}{2} \right) = \begin{bmatrix}
n_u \left( t - \frac{x_1}{v_f} \right) \\
n_d \left( t - \frac{x_d - x_1}{w_b} + k_j(x_d - x_1) \right) \\
n_u \left( t - \frac{x_1}{v_f} \right) \\
n_d \left( t - \frac{x_d - x_2}{w_b} + k_j(x_d - x_2) \right)
\end{bmatrix} + \alpha \Phi(\gamma) - \alpha' \Phi(\gamma) + \epsilon',
\]

where the error term \( \epsilon' \) is the combination error term, including the measurement error and estimation error of \( n(j' \tau, x_1) \) and \( n(j' \tau, x_2) \).

Unlike the standard linear mapping equation with a constant mapping matrix \( H \), the mapping coefficients \( \Phi(\gamma) \) and \( \Phi(\gamma') \) in Eqs. (23) and (26) are dependent on the prevailing traffic conditions on the boundary, namely, the difference between \( n_u \left( t - \frac{x}{v_f} \right) \) and \( n_d \left( t - \frac{x_d - x_1}{w_b} \right) \). Because the true values of cumulative flow counts are unknown, only the estimates of cumulative departure and arrival flow counts are available to calculate \( \Phi(\gamma) \) and \( \Phi(\gamma') \) when constructing the linear measurement equations. This possible estimation error, associated with the boundary cumulative flow counts, introduces one more source of error that should be included in the combined error terms \( \epsilon \) and \( \epsilon' \). On the other hand, as demonstrated in Eq. (21), when the standardized difference \( \gamma \) between \( n_u \left( t - \frac{xv_f}{n} \right) \) and \( n_d \left( t - \frac{xd-x_1}{w} \right) \), as shown in Eq. (19), is significantly large, the coefficients \( \Phi(\gamma) - \Phi(\gamma) \) and \( \Phi(\gamma') - \Phi(\gamma) \) take extreme values of 0 or 1, indicating that the internal condition at position \( (t, x) \) can be estimated directly from one of the forward vs. backward wave propagation procedures with high confidence levels.

4.2. Measurement equation for AVI data

In this subsection, we show that the proposed methodology can effectively incorporate the AVI (Bluetooth data) data source.

As illustrated in Fig. 3, two Bluetooth readers are separately located at the upstream and downstream locations. For a tagged vehicle, its passing time stamps at the two readers are denoted \( t \) and \( t + B_\tau \), respectively. To connect these samples with the cumulative vehicle counts at the both ends (i.e., unknown state variable in the freeway traffic state estimation problem), under a First-In-First-Out (FIFO) assumption for the three-detector model, we can
establish the following conditions to ensure that the tagged vehicle has the same cumulative flow count number when passing through both the upstream and downstream stations. Under an error-free environment, we have
\[ n_u(t) = n_d(t + B_t), \] (29)
while consideration of a combined error term \( \varepsilon'' \) leads to
\[ 0 = [1 -1] \times \begin{bmatrix} n_u(t) \\ n_d(t + B_t) \end{bmatrix} + \varepsilon'', \] where \( \varepsilon'' \sim \text{N}(0, R'') \),
and where \( R'' \) is the covariance of error term \( \varepsilon'' \).

The combined error term includes possible deviation in identifying \( n_u(t) \) and \( n_d(t + B_t) \). To calculate the error range in identifying \( n_u(t) \), we first denote \( \theta \) as a constant value for the likely feasible range of AVI readers’ clock drift errors and \( \bar{q}(t, x_u) \) as the average flow rate around time \( t \). Then, the standard deviation of the flow count deviation during a time duration of possible clock drifts is \( \theta \times \bar{q}(t, x_u) \). According to Eq. (15), we can further consider the estimation uncertainty of \( n_u(t) \) and \( n_d(t + B_t) \) (before incorporating AVI data) as \( \text{Var}_{n_u}(t, x_u) \) and \( \text{Var}_{n_d}(t + B_t, x_d) \). Thus, the variance \( R'' \) of the combined error can be approximated as
\[ R'' = (\theta \times \bar{q}(t, x_u))^2 + (\theta \times \bar{q}(t, x_u))^2 + \text{Var}_{n_u}(t, x_u) + \text{Var}_{n_d}(t + B_t, x_d). \] (30)

In this case, a linear measurement equation can be established as follows:
\[ Y'' = H''N'' + \varepsilon'', \] where \( \varepsilon'' \sim \text{N}(0, R'') \). (32)

Note that the measurement term in the above form is expressed as \( Y'' = 0 \) rather than the original passing time stamp samples. Additionally, the mapping vector \( H'' = [1 -1] \), and the system state vector \( N'' = [n_u(t) \quad n_d(t + B_t)]^T \). To consider AVI reader stations that are not located on the boundaries of segments, we can first map the passing time stamp measurements to the cumulative flow counts corresponding to the AVI reader locations, say, \( n(t, x_1) \) and \( n(t + B_t, x_2) \), where \( x_1 \) and \( x_2 \) are upstream and downstream locations of AVI readers. The second step is to connect \( n(t, x_1) \) and \( n(t + B_t, x_2) \) to the cumulative arrival and departure curves \( n_u(t) \) and \( n_d(t) \) at the boundary using the proposed stochastic three-detector model.

### 4.3. Measurement equation for GPS probe data

GPS probe data offer a semi-continuous trajectory of a vehicle in a segment. This section first extends the cumulative vehicle count-based approach in the previous section to construct measurement equations for each sample point along the trajectory. Second, we aim to use the local speed profile of the vehicle in our estimation framework.

**Vehicle Number Observations**

As shown in Fig. 3, a vehicle of number \( m \) traverses the segment along semi-continuous trajectory \( x(t + j''\Delta g, \forall j''=1,...,J'') \), where \( \Delta g \) denotes the sampling time interval of GPS, and \( J'' \) denotes total number of sampling points for an individual vehicle trajectory.

By applying the proposed STD model, we can map the cumulative vehicle count \( m \) at a sampling point with the following boundary conditions:
\[ m = n[t + j''\Delta g, x(t + j''\Delta g)] = V_u \Phi(\gamma) + V_d \Phi(-\gamma) + \alpha \phi(\gamma) + \varepsilon''', \] (33)
where the combined error term \( \varepsilon''' \) should include the following: (1) GPS location measurement errors; (2) the estimation error associated with the entry vehicle count \( m \); and (3) the estimation error of cumulative vehicle counts \( n[t + j\Delta g, x(t + j\Delta g)] \) through the proposed STD model. The second type of error range can be approximated using a similar formula for AVI data, i.e., \( \theta \times \bar{q}(t, x_u) \). According to Eq. (15), the variance of the third estimation error is \( \sigma^2 = v_2 - v_1^2 \).

Similar to the previous analysis, we can establish a linear measurement equation, shown below.
\[ Y''' = H'''N''' + \varepsilon''', \] where \( \varepsilon''' \sim \text{N}(0, R''') \), (35)
and where the transformed measurement term is \( Y''' = m - \Phi(-\gamma) \times k_j(x_d - x(t + j''\Delta g)) - \alpha \phi(\gamma) \), the system state vector
\[ N''' = \begin{bmatrix} n_u \left( t + j''\Delta g - \frac{x(t + j''\Delta g)}{v_f} \right) \\ n_d \left( t + j''\Delta g - \frac{x_d - x(t + j''\Delta g)}{w_b} \right) \end{bmatrix}. \]
Location-based speed samples

Typically, the location data of GPS probes are available second by second, and the adjacent locations of two sample points are used to compute the local speed measure. However, to reduce battery consumption and mitigate privacy concerns, some practical systems use a much longer time interval for data reporting, i.e., 30 seconds or 1 minute, while still sending local speed data (calculated from the internal second-by-second location data) to the data server.

![Speed-density relationship](image)

Fig. 4. Speed-density relationship.

To utilize the local speed measurement, we can convert local speed measurements into local density values. Fig. 4 shows the speed and density relationship. In the free-flow state, there are multiple density values corresponding to a constant free-flow speed, so one cannot deduce the unique density value in this case. On the other hand, during the congested state, because the vehicle-density relation is a monotonous curve, one can deduce the density from the speed measurement. By extending the measurement equation for local density in Eq. (28), we can incorporate the additional semi-continuous local speed data from GPS sensors.

5. Uncertainty quantification

5.1. Estimation Process using Kalman filtering

By considering the cumulative vehicle counts vector on the boundary as state vector $N$, we can apply a Kalman filtering framework to use the proposed linear measurement equations for each measurement type and obtain a final estimate of the boundary conditions. Specifically, given the prior estimate vector $N^-$ and the prior estimate error variance-covariance matrix $P^-$, the Kalman filter can derive the posterior estimate error variance-covariance $P^+$ and posterior estimate $N^+$ of $N$ using the following updated formula:

$$
N^+ = N^- + K(Y - HN^-);
$$

$$
P^+ = (1 - KH)P^-;
$$

where $K$ denotes the optimal Kalman filter gain factor:

$$
K = P^-H^T(HP^-H^T + R)^{-1}.
$$

When there are two sensors available on a single segment, one can directly use sensor data to construct the prior estimate vectors $N^-$ and $P^-$ through Eqs. (2-4). When there is only one sensor available on a segment, one must provide a rough guess of the unobserved boundary values, which leads to a much larger prior estimation error range for $P^-$. The proposed estimation framework uses cumulative flow counts as the state variable, which should be a non-decreasing time series at a certain location. Nevertheless, due to various sources of estimation errors, it is possible but less likely that the non-decreasing property of the estimated cumulative vehicle counts $n(t,x)$ does not hold, and the corresponding derived flow $n(t,x) - n(t - \Delta t, x)$ can be negative. A standard Kalman filtering framework, as described in Eqs. (36-38), does not consider inequality constraints. For simplicity, this study does not impose additional non-negativity constraints into the Kalman filtering framework to ensure that the derived flow is larger or greater than zero, and the negative flow volume can be easily corrected by a post-processing procedure. This post-
processing technique is also used in the general field of vehicle tracking, where a vehicle is typically moving forward, but the instantaneous speed might be estimated as negative due to various estimation errors.

In general, Kalman filtering is used in online recursive estimation and prediction applications. In this study, we focused on the offline traffic state estimation problem, and the Kalman filter was used as a generalized least squares estimator. Interested readers are referred to the dissertation by Ashok (1996) on the equivalence between these two estimators.

5.2 Quantifying the density estimation uncertainty and the value of information (VOI)

To evaluate the benefit of a possible sensor deployment strategy, we need to quantify the uncertainty reduction of the internal traffic state $n(t, x)$, which can be derived from the boundary conditions using the proposed STD model.

Furthermore, the density between intermediate position $x$ and $x + \Delta x$ at time $t$ can be directly calculated from cumulative counts $n(t, x)$:

$$k(t, x) = \frac{n(t, x) - n(t, x+\Delta x)}{\Delta x}.$$

According to Eqs. (14-15) in the proposed STD model, we can derive the mean and variance of the cumulative vehicle count estimates at any given location $x$ and time $t$. Let $E_k(t, x)$ and $\text{Var}_k(t, x)$ denote the mean and variance of density, respectively. First, we obtain

$$E_k(t, x) = \frac{E_n(t, x) - E_n(t, x+\Delta x)}{\Delta x}.$$

For simplicity, we can ignore the possible correlation between estimated adjacent cumulative flow counts and quantify the uncertainty associated with the density estimate as

$$\text{Var}_k(t, x) = \frac{\text{Var}_n(t, x) + \text{Var}_n(t, x+\Delta x)}{\Delta x^2}.$$

Similarly, we can derive the uncertainty measure for local flow rates. To estimate the uncertainty associated with local speed estimates, one can construct a linear mapping function between speed and density, as shown in the piecewise dashed line in Fig. 4, and then derive the speed estimation uncertainty as a function of the density estimation uncertainty.

To quantify the system-wide estimation uncertainty, one can simply tally the cell-based density estimation uncertainty across all cells on a segment and all simulation/modeling time intervals. Additional discussion on possible value of information measures in a Kalman filtering framework can be found in recent studies by Zhou and List (2010) on the origin-destination demand estimation problem, and Xing and Zhou (2011) on the path travel time estimation/prediction problem. Typically, when the total variance of traffic state estimation errors is smaller, the value of the information that can be obtained from the underlying sensor network is larger.

6. Numerical Experiments

In this study, we used a set of simulated experiments to investigate the performance of the proposed STD model on a 0.5-mile homogeneous segment with no entry or exit ramps, as shown in Fig. 5. The segment is divided into 10 sections, and the time of interest ranges from 0 to 1,200 s. Two loop detectors are installed at the upstream and downstream ends.

The other important parameters include a triangle-shaped flow-density relation, as shown in Fig. 6, where the free-flow speed $v_f = 60$ mile/h, the backward wave speed $w_b = 12$ mile/h, and the maximum density $k_f = 220$ veh/mile.
In this experiment, we consider a constant arriving flow rate \( u = 1,200 \text{ veh/h} \) \( t \in [0, 1,200] \), while the downstream bottleneck discharge rate \( \lambda \) is assumed to be time-dependent, i.e., \( \lambda(t) = \begin{cases} 600 \text{ veh/h} & t \in [0, 450] \\ 1,800 \text{ veh/h} & t \in [451, 1,200] \end{cases} \).

### 6.1 Estimations results of the STD model

Using the deterministic three-detector approach, the first step was to generate the ground truth boundary conditions in terms of deterministic arrival and departure cumulative vehicle count curves, as shown in Fig. 7. In particular, there are three shockwaves:

1. The first shockwave travels at a speed of 4 m/h, resulting in a long queue in the segment. When it finally spills back to the upstream site, the flow detected at the upstream sensor (compared to the actual arrival flow of 1,200 veh/h) is controlled by the bottleneck capacity of 600 veh/h.
2. The second backward recovery shockwave starts to propagate upstream at a speed of 12 m/h, right after the bottleneck capacity recovers to 1,800 veh/h at a time of 451 s.
3. The third shockwave is triggered by the transition where the arrival rate of 1,200 veh/h, starting at a time of 701 s, is lower than the normal bottleneck capacity.

The second step is to test the ability to capture the shockwave propagation using the proposed STD model. The corresponding stochastic boundary conditions, in terms of prior estimation cumulative vehicle counts vector \( N^- \) and a prior estimation variance-covariance matrix \( P^- \), were constructed under a sampling time interval \( \tau = 5 \text{ min} = 300 \text{ s} \), with a +/-10% measurement standard deviation. Based on \( N^- \) and \( P^- \), the STD model is able to produce the cell-based density estimates for all 10 sections inside the segment, shown in Fig. 8, and the corresponding uncertainty range for each cell in the space-time diagram, shown in Fig. 9.

As expected, Fig. 8 clearly shows the transition of the following four regimes: (1) free-flow (FF); (2) severe congestion (SC); (3) mild congestion (MC); and (4) free-flow. The boundaries of those regimes correspond to the underlying shockwaves.

To further demonstrate the computational details of the proposed STD model, let us consider a series of time stamp points at section 8, marked in Fig. 8. These seven points are numbered by time from 1 to 7, and each point corresponds to a particular traffic mode. Specifically, four points of interest, 1, 3, 5 and 7, are under the steady traffic state mode, and the other three points are in the transition boundaries. Table 2 shows the values of Clark’s approximation for estimating the mean and variance of the cumulative flow count using Eqs. (20-22).
Fig. 7. Arrival and departure cumulative vehicle count curves.

Fig. 8. The original cell based density estimation profile. Color denotes the density of each cell in veh/mile

Table 2
Values of Clark’s approximation under different traffic mode / transition

<table>
<thead>
<tr>
<th>Point</th>
<th>Traffic Mode/Transition</th>
<th>Time (min)</th>
<th>$\phi(0)$</th>
<th>$\phi(0.5)$</th>
<th>$\phi(-\gamma)$</th>
<th>$V_u$</th>
<th>$V_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>FF</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>33.33</td>
<td>45.83</td>
</tr>
<tr>
<td>2</td>
<td>FF→SC</td>
<td>3.3</td>
<td>0.3</td>
<td>0.5</td>
<td>0.5</td>
<td>58.33</td>
<td>58.33</td>
</tr>
<tr>
<td>3</td>
<td>SC</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>113.33</td>
<td>85.83</td>
</tr>
<tr>
<td>4</td>
<td>SC→MC</td>
<td>8.5</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>163.33</td>
<td>116.50</td>
</tr>
<tr>
<td>5</td>
<td>MC</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>205.17</td>
<td>191.50</td>
</tr>
<tr>
<td>6</td>
<td>MC→FF</td>
<td>13.2</td>
<td>0.8</td>
<td>0.5</td>
<td>0.5</td>
<td>257.00</td>
<td>257.00</td>
</tr>
<tr>
<td>7</td>
<td>FF</td>
<td>16</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>313.33</td>
<td>328.67</td>
</tr>
</tbody>
</table>
In Eqs.15-17 for generating the final cumulative flow count estimates, the cumulative normal distribution of the combined variable $\Phi(\gamma)$ and $\Phi(-\gamma)$ is the weights for forward wave vs. backward wave alternatives. Based on the numerical results in Table 2, we have the following interesting findings.

1. When the difference of systematic disutility, $V_u$ and $V_d$, is significantly large, the weight on each alternative, $\Phi(\gamma)$ and $\Phi(-\gamma)$, has an extreme value of zero or one, and the corresponding adjustment factor $\alpha \Phi(\gamma)$ is close to zero. It should be noted that, although Table 2 shows a value of zero for $\alpha \Phi(\gamma)$, it is actually a very small numerical value. By substituting $\alpha \Phi(\gamma)$=0 into the mean and variance estimation equation in Eqs. 16-17, we can verify that $E_n(t,x) = V_u$ and $Var_n(t,x) = \sigma^2_x$ for points 1 and 7, indicating that the uncertainty of the final estimate is controlled by the dominating alternative.

2. In cases of state transitions, i.e., free-flow to congested or congested to free-flow, $\Phi(\gamma)$ and $\Phi(-\gamma)$ stay at a level of 0.5, leading to almost equal weights for each alternative, and a positive adjustment factor $\alpha \Phi(\gamma)$ is needed. More interestingly, this case results in a large uncertainty or low confidence level about its exact value of the cumulative flow count, and the variance $Var_n(t,x)$ is jointly determined by both alternatives.

![Fig. 9. The original density uncertainty profile of cell based density estimation. Color denotes the estimated density variance of each cell.](image)

The overall uncertainty plot in Fig. 9 for each cell confirms our findings; that is, the boundaries of the state transition have large uncertainty. In addition, the estimation uncertainty generally increases when the time clock advances, as the measurement error in flow counts from the previous time intervals must be included in the cumulative flow count variable that appears later. Likewise, in Fig. 8, the contour of the shockwaves can be captured. Later, we compare this figure with a posterior density uncertainty profile to test the performance of a series of measurements.

6.2 VOI for heterogeneous measurements

From this point on, we are interested in the value of information of the following (additional) sensor network enhancement strategies:

1. providing a higher resolution for the existing boundary sensors by reducing the sampling time interval;
2. deploying an additional sensor between the original pair of sensors, at $x = 0.3$ miles (using vehicle count measurements, i.e., Eq. 26);
3. locating a pair of Bluetooth readers at the upstream and downstream boundaries, with a Bluetooth travel time reader measurement standard deviation $\theta = 5$ sec (using travel time measurements, i.e., Eq. 32);
4. equipping a certain percentage of vehicles with GPS mobile devices with a sampling time interval $\Delta t = 5$ sec along each probe vehicle trajectory (using vehicle number observations, i.e., Eq. 35).
Conceptually, these additional measurements are used to enhance the estimates and reduce the estimation uncertainty of cumulative vehicle counts at the upstream and downstream boundaries.

![Graph](image)

**Fig. 10.** Value of Information vs. sampling time interval. (a) existing sensors; (b) additional middle sensor.

Here, we adopt the density uncertainty to measure the VOI, which is defined as the inverse of sum of the estimated density variance of all cells. Fig. 10 displays the estimation performance improvement for the first two scenarios. Specifically, the VOI of the density estimation increases with a finer sampling time resolution of the existing sensors in the boundary. Keeping the same sampling resolution, the added middle sensor can produce additional VOI by an average of 10%.

![Graph](image)

**Fig. 11.** Comparable total uncertainty reduction curve for GPS and AVI in different market penetration rate

We then varied the market penetration rates from 10% to 90% for scenarios 3 and 4. As expected, the results shown in Fig. 11 indicate that both AVI and GPS measurements can significantly enhance the confidence level of the microscopic state estimation when the individual market penetration rate increases. Under the same market penetration rate of probe vehicles, the semi-continuous location-based samples from GPS sensors contribute more information than AVI readings, which are available only at the boundaries of the segment.

We now consider an integrated case with scenarios 2, 3 and 4 using the following settings: existing loop detectors at boundaries with a 5-min sampling time interval, an additional sensor at x = 0.3 miles with a 5-min sampling time interval, and a randomly selected portion of vehicles (10%) that are equipped with AVI Bluetooth and GPS sensors.
The proposed information-theoretic approach produces the posterior estimation cumulative vehicle counts vector \( N^+ \) and variance-covariance matrix \( P^+ \). Comparing the original estimated density uncertainty profile in Fig. 9 and Fig. 12 for the above integrated sensor network setting, we find that overall uncertainty has been dramatically reduced, but the cells corresponding to the back of the queue still have large uncertainty due to the inherent difficulty in estimating the exact probability of free-flow and congestion state between those state transition boundaries.

6.3 Preliminary discussions of modeling errors

The proposed model provides a theoretically rigorous mechanism for estimating internal traffic states on a freeway segment, but it is important to recognize possible modeling errors pertaining to the perfect triangular flow-density relationship, which is a key underlying assumption for both Newell’s kinematic wave model and the widely used CTM. By carefully examining our estimation results based on an NGSIM data set (FHWA, 2008), which provides detailed trajectory data based on video recordings at 0.1-s intervals, we identified the following possible modeling errors associated with the triangular flow-density relationship for both the deterministic and stochastic three-detector model.

1. Constant jam density \( k_j \). As an inverse of jam density, critical spacing could be much larger for trucks than for regular passenger vehicles. There are also significant variations in \( k_j \) depending on the driving conditions.

2. Variations in backward wave speed \( w_b \). Many studies (e.g., Kim and Zhang, 2008) have investigated stochasticity in \( w_b \).

3. Free-flow speed \( v_f \). Preferred free-flow speeds vary among individual drivers.

Particularly under congested conditions, the modeling errors in STD’s key formula \( z_d \left( t - \frac{L_D}{w_b} \right) + k_j L_D \) can be decomposed into the following elements.

1. Estimation errors in the boundary cumulative count \( z_d \left( t - \frac{L_D}{w_b} \right) \), which have been systematically addressed in this study.

2. Time index referencing errors in \(-\frac{L_D}{w_b}\). Let us denote \( w_b \) as the assumed backward wave speed in calculation and consider \( w_b' \) as the true backward wave speed. In this case, then the time index referencing error is \( \frac{L_D}{w_b'} - \frac{L_D}{w_b} \), which can further lead to the counting error of \( z_d \left( t - \frac{L_D}{w_b'} \right) - z_d \left( t - \frac{L_D}{w_b} \right) \).

3. Uncertainty and variations associated with \( k_j \). The assumed jam density value can lead to an adjustment factor error of \( k_j' L_D - k_j L_D \), where \( k_j' \) denotes the true jam density.
Likewise, under the free-flow condition, we can derive the modeling errors associated with variability of \( v_f \) in the first component \( z_u \left( t - \frac{x}{v_f} \right) \) of the minimization equation. Other error sources include the FIFO principle, which can be violated by complex lane changing behavior.

7. Conclusions

While there is a growing body of work on the estimation of traffic states from different sources of surveillance techniques, much of the prior work has focused on single representations, including loop detectors, GPS data, AVI tags, and other forms of vehicle tracking. This study investigated cumulative flow count-based system modeling methods that estimate macroscopic and microscopic traffic states with heterogeneous data sources on a freeway segment. Through a novel use of the multinomial probit model and Clark’s approximation method, we developed a stochastic three-detector model to estimate the mean and variance-covariance estimates of cumulative vehicle counts on both ends of a traffic segment, which are used as probabilistic inputs for estimating cell-based flow and density inside the space-time boundary and to construct a series of linear measurement equations within a Kalman filtering estimation framework. This study presented an information-theoretic approach to quantify the value of heterogeneous traffic measurements for specific fixed sensor location plans and market penetration rates of Bluetooth or GPS flow car data.

Further research will focus on the following three major aspects. First, the proposed single-segment-oriented methodology will be further extended for a corridor model with merges/diverges for possible medium-scale traffic state estimation applications. Second, the proposed model for the traffic state estimation problem can be further extended to a real-time recursive traffic state estimation and prediction framework involving multiple OD pairs with stochastic demand patterns or road capacities. Third, given the microscopic state estimation results, one can quantify the uncertainty of other quantities in many emerging transportation applications, e.g., fuel consumption and emissions that mainly dependent on cell-based or vehicle-based speed and acceleration measures; and link-based travel times that can be related to the cumulative vehicle counts on the boundary.

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