PLANNING-LEVEL METHODOLOGY FOR EVALUATING TRAVELER INFORMATION PROVISION STRATEGIES UNDER STOCHASTIC CAPACITY CONDITIONS

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Abstract

In this study, a non-linear optimization-based conceptual framework is proposed for incorporating stochastic capacity, travel time performance functions and varying degrees of traveler knowledge in an advanced traveler information provision environment. The proposed method categorizes commuters into two classes: (1) those with access to perfect traffic information every day, and (2) those with knowledge of the expected traffic conditions across different days. Using a gap function framework (for describing the user equilibrium under different information availability), a mathematical programming model is formulated to describe the route choice behavior of the perfect information (PI) and expected travel time (ETT) user classes under stochastic day-dependent travel time. Driven by an operational algorithm suitable for large-scale networks, the model was applied to a simple corridor and medium-scale networks to illustrate the effectiveness of traveler information under stochastic capacity conditions.

Keywords: stochastic road capacity, static traffic assignment, value of traveler information, route choice behavior
1. Introduction

Traffic congestion problems lead to a wide range of adverse consequences such as traffic delays, travel time unreliability, increased noise pollution as well as deterioration of air quality. Broadly speaking, traffic congestion occurs because the available capacity cannot serve the desired demand on a portion of the roadway at a particular time. Major sources of congestion include recurring bottlenecks, incidents, work zones, inclement weather, poor signal timing, special events and day-to-day fluctuations in normal traffic demand (Cambridge Systematics, 2005).

Considerable research efforts have been devoted to understanding and quantifying the effectiveness of different traffic mitigation strategies in addressing various sources of congestion. For instance, recurring congestion due to bottlenecks can be mitigated through road capacity enhancement, while real-time traffic information dissemination can reduce negative impacts of disruptions of non-recurring congestion due to traffic incidents and special events. The success of advanced traveler information systems deployment hinges on having an integrated system-level perspective, which calls for advanced transportation analysis tools to estimate how effective information provision and tolling strategies can encourage route/departure time/mode switching to more effectively utilize the remaining network capacity. This requires adopting and integrating various models that have evolved over the past decade, such as stochastic capacity analysis and dynamic traveler behavior modeling, within the classical user equilibrium analysis framework.

To evaluate traffic mobility impacts under user equilibrium conditions, a static traffic assignment approach is typically used for estimating link flows and travel times (Sheffi, 1985). Conventional static traffic assignment approaches generally assume users have perfect knowledge of network conditions, and that link capacities are fixed or deterministic. However, this perfect information assumption does not allow modelers to realistically evaluate the longer-term benefits of real-time traveler information provision strategies. Recent empirical research (Brilon, et al. 2005, 2007) indicates that highway capacity can be characterized as a random variable. Even under constant geometric, traffic, environmental, and/or operational conditions, road capacities vary with time over a certain range around a mean value. More precisely, highway capacity is the result of complex driver behavioral interaction. It also varies according to many external factors such as accidents, incidents, severe weather, or work zones.

In the field of highway capacity research, stochastic link capacity can be defined as the maximum sustainable flow rate, below which vehicles reasonably can be expected to freely traverse a roadway during a specified time period, and above which unacceptable travel conditions prevail. The inclusion of stochastic capacity at the critical points in a highway network such as freeway bottlenecks and signalized intersections produces a more realistic modeling of travel time variability and introduces the concept of sustainable flow rates. In fact, variability in the transportation system cannot be fully considered and the true value of traveler information provision cannot be accurately estimated without accounting for this critical source of unreliability (namely, stochastic capacity).

In addition to stochastic capacity four other sources also contribute to increased travel time variability and unreliability: 1) Stochastic input demand 2) Random departure time choice and route choice which can lead to uncertain flow inputs for a certain set of links (Noland and Polak, 2002). 3) The absence of precise and real-time traffic information, due to sensor coverage or limited traveler knowledge/experience, which can further compound the issue of travel time uncertainty. In fact, there are only a small fraction of travelers who currently have full access or are willing to always retrieve pre-trip or en-route traveler information through web-based traveler information sites, car radio, dynamic message signs or Internet-connected navigation devices (Khattak et al. 2008). When making route choices, the majority of travelers still rely on their personal knowledge and driving experiences that have been gained over a long time period of time, which can be described as the expected travel time (caused by stochastic demand and capacity). When there is a significant variation in capacity, the resulting network conditions could deviate considerably from the average traffic pattern. In this case, the expected value-based travel knowledge should be treated as a biased estimate to the current traffic state. 4) Traveler perception error which is typically modeled in a stochastic traffic assignment framework to capture unbiased random noises (with a
mean of zero) associated with drivers’ socio-economic characteristics, personal observations, as well as the quality of traveler information.

Many studies in the literature (e.g. Yang, 1998; Yang and Meng, 2001; Yin and Yang, 2003) use a general stochastic user equilibrium (SUE) traffic assignment model to quantify the value of traveler information under deterministic and time-invariant road capacity, where all travelers are assumed to have unbiased travel time estimates but with different degrees of uncertainty with respect to their own information user group (e.g. equipped with an ATIS or not). The uncertainty levels are modeled through the perception error term in SUE. That is, commuters with advanced traveler information have smaller perception errors, compared to travelers without access to ATIS channels. Recently, in order to reformulate the traditional static traffic assignment problem under stochastic capacity conditions, different numerical approximation methods have been proposed to describe the travel time variability for a *single-day* traffic equilibrium solution. For example, Lo and Tung (2003) and Lo et al. (2006) adopted Mellin transforms to describe network performance caused by stochastic link capacities. Ng and Waller (2010) presented a fast Fourier transform-based method to characterize travel time variability distributions due to random capacity. It should be noticed that, in the context of ATIS benefit evaluation, the single-day steady-state representation is insufficient to capture the day-varying route choice behaviors by ATIS equipped drivers.

In order to distinguish different system throughput states (i.e. stochastic capacity) in a user equilibrium framework, De Palma and Picard (2005) used a graphical method to consider two types of information user classes, including (1) those with perfect information on good days and bad days (i.e. under normal and reduced capacity); and (2) those with information on expected travel times on different days. Their pioneering investigation provides great theoretical insights into analyzing traveler behavior under stochastic capacity. Along these lines, this research has focused on developing a rigorous mathematical programming model and efficient solution algorithms for general traffic networks with continuous stochastic capacity distributions.

With a special focus on formulating and solving the steady-state user equilibrium problem with stochastic capacity, this research aims to find a network flow pattern that satisfies a generalization of Wardrop’s first principle: travelers with the same origin-destination pair experience the same and minimum expected travel time along any used paths on different days, with no unused path offering a shorter expected travel time. Based on the classical gap function-based framework for user equilibrium by Smith (1993), Lo and Chen (2000) reformulated the nonlinear complementarity problem for traffic user equilibrium with fixed demand and fixed capacity to a route flow-based mathematical program through a convex and smooth gap function. A recent study by Lu et al. (2009) further extended Lo and Chen’s model to general dynamic traffic networks with a route swapping rule, which is based on a first-order gradient descent algorithm for solving convex optimization problems. Through the gap function-based reformulation for user equilibrium, the proposed model explicitly considers the stochastic nature of network capacity over different days and represents travelers’ imperfect information and general knowledge about the random travel time variations. A solution method is developed to find the equilibrium path flow distribution, while stochastic travel times on different days are generated from a sampling-based simulation framework.

### 2. Conceptual framework

The conceptual modeling framework is illustrated in Fig. 1 using a simple corridor with a single origin-to-destination pair and two paths $p=1$ for the primary path, $p=2$ for the alternative path, where $p$ is the path index. As each path only has one link, path 1 is denoted as link $a=1$ with a free-flow travel time of 20 minutes, and path 2 is denoted as link $a=2$ with a free-flow travel time of 30 minutes, where $a$ is the link index. This example considers five different days $d = 1, 2, 3, 4$ and 5, and the peak hour demand is $Q=8000$ vehicles per hour on each day.

Following a similar analysis setting in the study by De Palma and Picard (2005), the first illustrative example considers day 1 as the “bad” day on path 1, with a reduced capacity for the primary route, and days 2, 3, 4, and 5 as good days with the full capacity available. As detailed in Table 1, the primary path has the following capacity values:
On the bad day ($d=1$) it is 3,000 vehicles per hour (vph) per link.

On the good days ($d=2, 3, 4, 5$) it is 4,500 vph per link.

The alternative path is assumed to have a fixed capacity of $c_{a,d} = 3,000$ on days $d=1, 2, 3, 4$ and 5, where $c_{a,d}$ is defined as the capacity of link $a$ on day $d$.

Free-flow travel time = 20 min
Capacity = 4500 (veh/h) on good days
or 3000 (veh/h) on bad days

Primary path: 1
Alternative path: 2
Free-flow travel time = 30 min
Capacity = 3000 (veh/h) on all days

Fig. 1. Simple network used as an illustrative example of the framework

Table 1 Day-dependent path demand, capacity and travel time values

<table>
<thead>
<tr>
<th>Day-Dependent Capacity</th>
<th>Path 1</th>
<th>Path 2</th>
<th>Path 1</th>
<th>Path 2</th>
<th>Path 1</th>
<th>Path 2</th>
<th>Path 1</th>
<th>Path 2</th>
<th>Path 1</th>
<th>Path 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily Capacity on Path/Link 1 (veh/h) $C_{a=1,d}$</td>
<td>3000</td>
<td>3000</td>
<td>4500</td>
<td>3000</td>
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<td>3000</td>
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</tr>
<tr>
<td>Daily Capacity on Path/Link 2 (veh/h) $C_{a=2,d}$</td>
<td>4500</td>
<td>4500</td>
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<tr>
<td>PI-Based User Equilibrium</td>
<td>46.36</td>
<td>33.64</td>
<td>61.72</td>
<td>18.28</td>
<td>61.72</td>
<td>18.28</td>
<td>61.72</td>
<td>18.28</td>
<td>61.72</td>
<td>18.28</td>
</tr>
<tr>
<td>Flow (Veh/hour/link)</td>
<td>55.03</td>
<td>55.03</td>
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<tr>
<td>Travel Time (min)</td>
<td>37.1</td>
<td>37.1</td>
<td>30.6</td>
<td>30.6</td>
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</tr>
<tr>
<td>ETT-Based User Equilibrium</td>
<td>54.0</td>
<td>32.2</td>
<td>26.7</td>
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<tr>
<td>Flow (Veh/hour/link)</td>
<td>2497</td>
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<tr>
<td>Travel Time (min)</td>
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*reduced capacity

To setup a mathematical programming model for steady-state traffic equilibrium, the non-negative flow variables $f_{p,d}$ is considered as the traffic flow using path $p$ on day $d$. Obviously, the path flow distribution should ensure the total demand constraint on each day:

$$f_{1,d} + f_{2,d} = Q \quad \forall d$$

Let $T_{p,d}$ be defined as the travel time on path $p$ on day $d$, which can be calculated from the BPR function such as

$$T_{p,d} = FTT_a \times \left[1 + \alpha \times \left(\frac{f_{a,d}}{c_{a,d}}\right)^\beta\right]$$

3
where $FTT_a$ is the free-flow travel time of link $a$. Coefficients $\alpha$ and $\beta$ are set to commonly used default values 0.15 and 4, respectively.

Now the two different degrees of traveler knowledge can be examined.

**Perfect Information (PI) based user equilibrium**

Every day, perfect travel time estimates (i.e. zero prediction error) for all links are available to travelers to make route decisions, and travelers can switch routes every day. This perfect and complete information assumption for each day is consistent with deterministic static traffic assignment, which usually considers a typical weekday. It should be cautioned that this assumption might not be realistic from a dynamic traffic assignment perspective, as both pre-trip and en-route traveler information available to commuters are essentially forecasted estimates of traffic conditions unfolding in the future, with always some degree of prediction errors. According to Wardrop’s first principle of user equilibrium, for a specific origin-destination pair, travelers with perfect information experience the same and minimum travel time along any used paths on each day $d$, with no unused path offering a shorter travel time.

To construct the objective function in the optimization model, the following gap function (for each day $d$) can be used to characterize the Karush-Kuhn-Tucker optimality conditions (Wiki, 2010) required for reaching the user equilibrium for perfect information users.

$$gap_{d}^{PI} = f_{1,d}^{PI} \times (T_{1,d} - \pi_{d}) + f_{2,d}^{PI} \times (T_{2,d} - \pi_{d}) = 0, \forall d$$

(3)

Where $f_{1,d}^{PI}$ and $f_{2,d}^{PI}$ are path flow rate of PI users on paths 1 and 2, respectively, on day $d$, where $\pi_{d}$ is minimum path travel time on day $d$

$$\pi_{d} = \min(T_{1,d}, T_{2,d}), \forall d$$

(4)

For the illustrative simple corridor, Table 1 shows the traffic assignment results when all travelers in the network have access to perfect information, and a standard deterministic user equilibrium state is reached every day. See also Fig. 2 for a graphical representation of the flows and travel times for PI users.

**Expected travel time (ETT) knowledge-based user equilibrium**

As there are different realized capacity values on different days, the travel times on different links can be viewed as a set of random variables. In reality, most travelers are not equipped with advanced traveler information systems, so they rely on their expected travel times (based on their knowledge and experience) over different days to make route choices. The expected travel time can be considered as the long-run average, or more precisely, the probability-weighted sum of the possible travel time values from different days. Under a user equilibrium condition with ETT users, the expected travel times on used routes in the network are assumed to be the same, and accordingly, an ETT user selects the same route every day, regardless the actual traffic conditions.

The expected travel time for link $a$ with random capacity $c_a$ over different days can be represented as

$$\bar{T}_a(f_a, c_a) = \frac{\sum_{d} T_{a,d}(f_{a,d}, c_{a,d})}{|d|},$$

(5)

Where travel time on each day $d$ for link $a$ $T_{a,d}(f_{a,d}, c_{a,d})$ is a function of the prevailing flow and capacity on that particular day. For link $a=1$ in the illustrative example,
Fig. 2. Equilibrium solutions with 100% PI users.  
Point E: reduced-capacity days, equilibrated travel times = 37.1 min, 4636 vehicles on link 1 and 3364 vehicles on link 2; 
Point F: full-capacity days, equilibrated travel times = 30.6 min with 6172 vehicles on link 1 and 1828 vehicles on link 2.

\[
\bar{T}_a(f_a, c_a) = 0.2 \times \bar{T}_a(f_a, c_a^R) + 0.8 \times \bar{T}_a(f_a, c_a^F)
\]

\[
= 0.2 \times FFTT_a \times \left(1 + \alpha \times \left[\frac{f_a}{c_a^R}\right]^{\beta}\right) + 0.8 \times FFTT_a \times \left(1 + \alpha \times \left[\frac{f_a}{c_a^F}\right]^{\beta}\right)
\]

(6)

\[
= FFTT_a \times \left(1 + 0.2 \times \alpha \times \left[\frac{f_a}{c_a^R}\right]^{\beta} + 0.8 \times \alpha \times \left[\frac{f_a}{c_a^F}\right]^{\beta}\right)
\]

Where \(c_a^R\) and \(c_a^F\) corresponds to the reduced and full capacity on link \(a\).

Note that \(\bar{T}_a\) is different from the expected value (EV) solution \(T_a(f_a, \bar{c}_a)\) typically used in the context of stochastic optimization, which can be calculated using the expected value of capacity on link \(a\), \(\bar{c}_a\).

\[
T_a(f_a, \bar{c}_a) = FFTT_a \times \left(1 + \alpha \times \left[\frac{f_a}{\bar{c}_a}\right]^{\beta}\right) = FFTT_a \times \left(1 + \alpha \times \left[\frac{f_a}{0.2 \times c_a^R + 0.8 \times c_a^F}\right]^{\beta}\right)
\]

(7)
In this study, we generalize Wardrop’s first principle to describe the equilibrium conditions for travelers relying on their expected travel time to make route decisions: travelers with the same origin-destination pair experience the same and minimum expected travel time along any used paths on different days, with no unused path offering a shorter expected travel time. Obviously, when there is a single capacity value, then the above conditions are consistent with the standard user equilibrium with deterministic capacity, as the expected travel time devolves to the travel time on the single day.

The corresponding KKT condition can be re-written as

\[ \text{gap}^{\text{ETT}} = f_1^{\text{ETT}} \times (\bar{T}_1 - \bar{\pi}) + f_2^{\text{ETT}} \times (\bar{T}_2 - \bar{\pi}) = 0 \]  

where \( \bar{\pi} \) is the least expected travel time between the given OD pair over a multi-day horizon satisfies

\[ \bar{\pi} = \min(\bar{T}_1, \bar{T}_2) \]  

An ETT knowledge-user uses the same route on different days, which leads to a day-invariant ETT flow pattern:

\[ f_a^{\text{ETT}} = f_{a,d-1}^{\text{ETT}} = f_{a,d-2}^{\text{ETT}} = f_{a,d-3}^{\text{ETT}} = f_{a,d-4}^{\text{ETT}} = f_{a,d-5}^{\text{ETT}} \quad \forall a \]  

When \( \text{gap}^{\text{ETT}} = 0 \), it can be shown that if \( f_a^{\text{ETT}} > 0 \), then \( \bar{T}_a = \bar{\pi} \). That is, the selected routes by expected travel time information users between an OD pair have equal and minimum costs. On the other hand, if \( f_a^{\text{ETT}} = 0 \), then \( \bar{T}_a \geq \bar{\pi} \), which indicates that all unused routes by ETT users have greater or equal costs (compared to the used path costs). These two conditions further imply that no individual trip maker with expected travel time information can reduce his/her expected path costs by switching routes on any given day, under a user equilibrium condition.

![Diagram](image.png)

**Fig. 3.** Solutions with 100% ETT information users, the expected travel time function (TTF) is generated by assigning a 20% weight to TTF with reduced-capacity (RC) days and an 80% weight to TTF with full-capacity (FC). The ETT-based user equilibrium corresponds to the intersection (in orange) of expected TTF on path 2 and path 1. 5503 vehicles are using link 1 and 2497 vehicles are using link 2 each day.

- **Point A:** travel time = 54.0 min on link 1, reduced-capacity days
- **Point B:** travel time = 26.7 min on link 1, full-capacity days
Point G: travel time = 32.2 min on link 2 every day, and the expected travel time on link 1 is the same 32.2 min.

If it is assumed that all users rely on ETT information in the simple corridor, then the ETT-based user equilibrium assigns about 5503 vehicles on path 1, and about 2497 vehicles on path 2, leading to different travel time on 5 different days shown in Table 1. As both paths carry positive flows, their average travel times over the 5-day horizon are the same at 32.2 min.

Quantification of the value of information

In Fig. 3 and Fig. 4, the relative travel time savings were examined under different market penetration rates of perfect information users when there are both PI and ETT users. Denote the amount of PI users as \( f^{PI} \), and denote the flow volume at points A, B, C, D and E as \( f^A, f^B, f^C, f^D, f^E \). Obviously, \( f^A = f^B \).

If \( f^{PI} \) is less than \( f^A - f^E \), then those PI travelers enjoy a travel time saving from point C to point D with

\[
T_{eq}(f^A - f^{PI}, c_{eq}) - T_{eq}(Q - f^A + f^{PI}, c_{eq})
\]

The travel time saving \( (T_C - T_D) \) diminishes as the flow of PI users increases, where \( T_C \) and \( T_D \) correspond to the travel time at points C and D. When \( f^{PI} \) further increases and reaches the value \( f^A - f^E \), both paths have the same equilibrated travel time of 37.1 min. If \( f^{PI} \) exceeds \( f^A - f^E \), none of the PI users is able to reduce his/her travel time by switching routes and the equilibrium point remains the same as the equilibrium point (E) for 100% PI users with reduced capacity shown in Fig. 5.

![Travel Time Function on Path 2](image)

**Fig. 4. Solutions on a reduced-capacity day.**
5% PI users (400 vehicles) and 95% ETT users (7600 vehicles),
Point C5: 5284 ETT vehicles on link 1, travel time = 48.9 min.
Point D5: 2716 vehicles (2316 ETT users + 400 PI users) on link 2, travel time = 33.0 min.
PI users travel time saving = 44.0 - 33.0 = 11.0 min, where 44.0 min is the average travel time for ETT users = (5284*48.9 + 2316*33.0)/7600.

10% PI users (800 vehicles) and 90% ETT users (7200 vehicles)
Point C10: 5060 ETT vehicles on link 1, travel time = 44.3 min
Point D10: 2940 vehicles (2140 ETT users + 800 PI users) on link 2, travel time = 34.1 min,
PI users travel time saving = 41.3 - 34.1 = 7.1 min, where 43.1 min is average travel time for ETT users

20% PI users (1600 vehicles) and 80% ETT users (6400 vehicles)
Point E20: 64 PI, 4572 ETT, and 4636 vehicles in total on link 1, travel time = 37.1 min
1536 PI, 1828 ETT, and 3364 vehicles in total on link 2, travel time = 37.1 min
PI users travel time saving = 0 min

As the proposed model approximates a long-term stochastic steady state under stochastic capacity, Fig. 5 further shows a possible sequence of the corresponding travel times on 2 different routes over a 20-day horizon. Note that, even though there still exists a 5-day cycle with 4 good days and 1 bad day with reduced capacity, the impaired capacity conditions occur on day 1, day 7, day 11 and day 19 in this example. This irregularity, which is permitted by the model, shows the unpredictability of stochastic travel times, so the ETT knowledge users simply consider the average travel time on path 1 (with a 20% chance or risk of severe traffic congestion) in their long-term route choice decisions. It should also be noted that the behavioral model used here is structurally different from the commonly used day-to-day learning model in a DTA framework, even though the underlying traffic states are represented within the same multi-day structure. In a day-to-day dynamic learning model, the perceived travel time on day \( d + 1 \) is updated using experienced travel times from previous days \( d, d - 1, d - 2 \) and so on, and the path can also be changed on a daily basis. In comparison, the ETT knowledge users consider average traffic conditions over all the days as a whole, and always stick to the same route.

![Travel Time on Path 1 and Path 2](image)

**Fig. 5.** Day-dependent travel times on different routes.

3. General mathematical problem formulation

This section extends the above conceptual framework to a general network with multiple origin-destination pairs and with road pricing strategies.
Formulation
The sets and subscripts, parameters and decision variables in the proposed flow assignment model are introduced as follows:
Indices:
\(i\) = index of origins, \(i = 1, \ldots, I\), where \(I\) is the number of origins
\(j\) = index of destinations, \(j = 1, \ldots, J\), where \(J\) is the number of destinations
\(p\) = index of paths, \(p=1, \ldots, P\), where \(P\) is the number of paths between an OD pair \(i\) and \(j\)
\(a\) = index of links, \(a=1, \ldots, A\), where \(A\) is the number of links in networks
\(d\) = index of days, \(d=1, \ldots, D\), where \(D\) is the number of days over analysis horizon

Input Parameters:
\(c_{a,d}\) = capacity of link \(a\) on day \(d\)
\(s_{a,d}\) = toll value charged on link \(a\) on day \(d\)
\(q_{i,j}\) = OD demand volume between an OD pair \((i,j)\)
\(\delta_{p,a}\) = path-link incidence coefficient, \(\delta_{p,a}=1\), if path \(p\) passes through link \(a\), and 0 otherwise
\(\gamma\) = market penetration rate of the perfect information (PI) users as a function of the total OD demand
\(VOT\) = value of time in dollars per minute

Decision variables:
\(f_{PI, p,d}^{i,j}\) = flow of PI users on path \(p\) for OD pair \((i,j)\) on day \(d\)
\(f_{ETT, p,d}^{i,j}\) = flow of ETT users on path \(p\) for OD pair \((i,j)\) (flow rates are the same across different days)
\(V_{a,d}\) = total flow on link \(a\) on day \(d\)
\(T_{a,d}\) = travel time on link \(a\) on day \(d\)
\(U_{a,d}\) = generalized disutility on link \(a\) on day \(d\), which is a function of capacity \(c_{a,d}\) and link flow
\(U_{p,d}^{i,j}\) = generalized disutility of path \(p\) between OD pair \((i,j)\) on day \(d\)
\(\bar{U}_{p}^{i,j}\) = expected disutility of path \(p\) between OD pair \((i,j)\) over the multi-day horizon
\(\pi_{d}^{i,j}\) = day-dependent least path disutility between OD pair \((i,j)\) on day \(d\)
\(\bar{\pi}^{i,j}\) = least expected disutility between OD pair \((i,j)\) over the multi-day horizon

The proposed model incorporates the two user classes into a static traffic assignment framework under stochastic capacity that varies on a daily basis during the peak hour. The objective function aims to minimize the total gap for users with perfect traffic information and users with imperfect information based on expected travel times.

Objective function:
\[
\min \text{Gap} = \sum_d \sum_i \sum_j \sum_p \left[ f_{p,d}^{\text{PI},i,j} \times (U_{p,d}^{i,j} - \pi_{d}^{i,j}) + f_{p}^{\text{ETT},i,j} \times (\bar{U}_{p}^{i,j} - \bar{\pi}_{d}^{i,j}) \right]
\]  \hspace{1cm} (11)

**PI flow constraints:**
\[\gamma \times q_{i,j} = \sum_p f_{p,d}^{\text{PI},i,j} \quad \forall i, j, d\]  \hspace{1cm} (12)

**ETT flow constraints**
\[(1 - \gamma) \times q_{i,j} = \sum_p f_{p}^{\text{ETT},i,j} \quad \forall i, j\]  \hspace{1cm} (13)

**Path - link flow balance constraints**
\[v_{a,d} = \sum_i \sum_j \sum_p \left( f_{p,d}^{\text{PI},i,j} \cdot \delta_{p,a} \right) + \sum_i \sum_j \sum_p \left( f_{p}^{\text{ETT},i,j} \cdot \delta_{p,a} \right) \quad \forall a, d\]  \hspace{1cm} (14)

**Path - link cost connection**
\[U_{a,d} = T_{a,d}(v_{a,d}, c_{a,d}) + \frac{s_{a,d}}{VOT} \quad \forall a, d\]  \hspace{1cm} (15)

\[U_{p,d}^{i,j} = \sum_a \left( U_{a,d} \cdot \delta_{p,a} \right) \quad \forall i, j, d, p\]  \hspace{1cm} (16)

**Average disutility definitional constraint:**
\[\bar{U}_{p}^{i,j} = \frac{1}{D} \sum_d U_{p,d}^{i,j} \quad \forall i, j, p\]  \hspace{1cm} (17)

**Least disutility definitional constraints:**
\[\pi_{d}^{i,j} \leq U_{p,d}^{i,j} \quad \forall i, j, p, d\]  \hspace{1cm} (18)

\[\bar{\pi}_{d}^{i,j} \leq \bar{U}_{p}^{i,j} \quad \forall i, j, p\]  \hspace{1cm} (19)

Constraints (12) and (13) show the relationship between OD demand and path flows for each information class. Eq. (14) aggregates path flows from two different user classes to link flows. Eqs. (15-16) calculate the path disutility for each path on day \(d\), where the dollar value of road toll is incorporated as equivalent travel time through value of time (VOT). Eq. (17) defines the average disutility for each path across different days, which will be used in the gap function for ETT users in objective function (11).

For comparison purposes, the system optimal benchmark can also be defined as:
\[\min z = \sum_d \sum_a \left[ v_{a,d} \times T_{a,d}(v_{a,d}, c_{a,d}) \right]\]  \hspace{1cm} (20)

where the flow to be optimized \(f_{a,d}\) can be day-varying.

Because the path utility function is convex and monotonic with respect to path flow and the expected travel time function is a convex combination of day-dependent travel times, the resulting gap function can be shown to be smooth, convex and bounded. Interested readers are referred to the paper by Lo and Chen (2000) for the detailed proof on a similar reformulation. These nice features allow a wide range of efficient nonlinear programming solution algorithms, such as gradient projection algorithms, to be applied to solve the proposed model.

**Spreadsheet tool for calculating multi-day user equilibrium**

Considering the above simple corridor with two routes, a spreadsheet application is developed with the following input elements: (1) deterministic, fixed demand, (2) day-dependent capacity generated from a stochastic capacity distribution, and (3) static travel time functions. The proposed non-linear model is formulated using the embedded optimization solver in Excel to bring the total gap function toward zero, while the traffic flow of PI and ETT users on different routes are considered as variables to be optimized (i.e. cells to be changed). A wide range of travel time statistics can be derived from the traffic assignment
results, such as the mean and variance of day-dependent travel times. The spreadsheet tool can be downloaded at www.civil.utah.edu/~zhou/traffic-equilibrium-under-stochastic-capacity.xlsx.

Fig. 4 demonstrates how the spreadsheet model is operated according to the following steps:

1. **Input data preparation.** Prepare the following input data:
   a. total demand;
   b. PI market penetration γ;
   c. random capacity on different days; and
   d. BPR step function parameters

2. **Volume calculation:** Corresponding to Eq. (1), the hourly traffic volume is distributed to different paths in block E1 and E2, which also contain variables to be optimized. The total hourly traffic volume is calculated paths in block E3.

3. **Link travel time calculation:** The day-dependent link travel times and the expected travel times are calculated using Eqs. (2) and (5) in block F, based on the link flow from block E1 and E2.

4. **Minimum travel time determination:** Find the minimum travel time πₜ on each day and π for the average travel time in block G.

5. **Defining the objective function:** Define the objective function related to the gap functions in block H, corresponding to Eqs. (3) or (9) for PI and ETT users, respectively.

6. **Solving the optimization model:** Solve the optimization model and derive travel time statistics according to travel time data in block F.

**Fig. 6. Spreadsheet-based calculation model.**

**4. Solution algorithm**

The solution algorithm executing the above steps is depicted in Fig. 7.
1. Generate stochastic capacity $C_d$ for all links on day $d=1,2,\ldots,D$

2. Assign all vehicles of each OD pair to the shortest path

3. Perform multi-day traffic simulation runs

4. Find descent direction

5. Path Assignment

6. Link flow aggregation

7. Check convergence using gap functions

Fig. 7: Solution algorithm for static traffic assignment with both PI and ETT users

In order to iteratively reduce the overall gap in the proposed optimization problem for a general network with multiple origins and destinations, we extend a descent search solution framework developed by Lu et al. (2009), which also used a path-based gap function to describe the dynamic traffic equilibrium pattern. Fig. 7 presents the iterative procedure for solving the multi-class static traveler assignment problem under stochastic capacity conditions. The proposed procedure adds day-dependent simulation, path finding and
assignment dimensions to the existing static traffic assignment algorithm that typically assumes deterministic road capacity conditions. In this study, we implement the proposed algorithm within a mesoscopic traffic assignment framework, which represents flow as vehicles with origin, destination and path attributes. Recall that, in conventional assignment programs, a vehicle is associated with a single path. In the proposed multi-day traffic assignment algorithm, an ETT vehicle still follows a single path across different days, but a PI vehicle can use and store different (day-dependent) paths on different days.

The main steps of the solution procedure are described as follows:

**Step 1: Day-dependent capacity generation.**
Generate road capacity vector \( C_d = [c_{a,d}] \), for all link \( a=1, 2, \ldots, A \), on day \( d=1, 2, \ldots, D \), according to given stochastic capacity distributions.

**Step 2: Initialization.**
Start with iteration number \( n=0 \). Generate PI and ETT vehicles according to given market penetration rate \( \gamma \). For each OD pair, compute the shortest path (in distance) and assign both PI and ETT vehicles to the corresponding shortest path.

**Step 3: Multi-day traffic simulation with stochastic capacity.**
On each day \( d=1, 2, \ldots, D \), for given link flow patterns, generate day-dependent link travel times according to stochastic capacity vector \( C_d \). The simulation results generate link travel time \( T_{a,d} \) for link \( a=1, 2, \ldots, A \), on day \( d=1, 2, \ldots, D \).

**Step 4: Find descent directions for traffic assignment**
Find the Least Travel time Path (LTP) using day-dependent link travel time \( T_{a,d} \) on each day \( d \), for link \( a=1, 2, \ldots, A \).

Find the Least Expected Travel time Path (LETP) using average link travel time \( \bar{T}_a = \frac{\sum T_{a,d}}{D} \), for link \( a=1, 2, \ldots, A \).

**Step 5: Path assignment for PI and ETT vehicles**
For each day \( d \), a certain percentage of PI vehicles are assigned to the least travel time path. By adapting the path-swapping method proposed by Lu et al. (2009), this study uses the following probabilistic ratio for a vehicle on path \( p \) to switch to the least travel time path at iteration \( n \):

\[
\frac{1}{n+1} \times \frac{U^{i,j}_{p,d} - \pi^{i,j}_d}{U^{i,j}_{p,d}}
\]

The first term \( 1/(n+1) \) is equivalent to the fixed step size in the Method of Successive Average (MSA). The second term ensures that the path swapping probability is proportional to the relative difference between the experienced path travel time \( U^{i,j}_{p,d} \) and the minimum path travel time \( \pi^{i,j}_d \). An intuitive interpretation for this heuristic swapping rule is that, travelers on longer paths (i.e. farther from the equilibrium solution) are more likely to switch to the least travel time path than those on paths with travel cost closer to the least travel time path.

Similarly, a certain percentage of ETT vehicles are swapped to the least expected travel time path, the route swapping probability at iteration \( n \) can be determined by

\[
\frac{1}{n+1} \times \frac{\bar{U}^{i,j}_{p} - \bar{\pi}^{i,j}}{\bar{U}^{i,j}_{p}}
\]

As shown in Lu et al. (2009), search directions specified by Eqs. (21-22) can be proven to be in the descent direction of the gap function in Eq. (11) for \( f_{p,d}^{PI,i,j} \), \( f_{p}^{ETT,i,j} \), respectively, at iteration \( n \).

**Step 6: Link flow aggregation**
For each day \( d \), calculate the aggregated link volume \( v_{a,d} \) using PI flow volume on day \( d \) and ETT flow (across every day), using Eq. (14).
Step 7: Convergence checking
Calculate the gap function as shown in Eq. (11), if $\text{Gap} < \delta$ convergence is achieved, where $\delta$ is a pre-specified parameter. If convergence is attained, stop. Otherwise, go to Step 3.

5. Experimental results

The first set of experiments uses the simple corridor with two routes in the illustrative example, shown in Table 1. In this set of experiments, it is further assumed that links 1 and 2 have 3 and 2 lanes, respectively, and on this basis 100 days of random lane capacity are generated. The headway data used in this analysis were obtained from a recent research effort by Jia et al (2010). In their study, the pre-breakdown time headways are found to follow a shifted log-normal distribution with the following probability density function:

$$f_X(x; \mu, \sigma) = \frac{1}{(x-c)\sigma\sqrt{2\pi}} e^{-\frac{(\ln(x-c)-\mu)^2}{2\sigma^2}}, \quad x > 0$$

(23)

Where,
- $x$ is the average pre-breakdown headway (in second) for 15-minute interval,
- $c$ is the minimum pre-breakdown headway (in second),
- $\mu$ is the mean of the variable's natural logarithm, and
- $\sigma$ is the standard deviation of the variable's natural logarithm.

Their calibration results based on data from several bottleneck locations in the Bay Area, California show that $c=1.5$ seconds, $\mu=-0.97$, and $\sigma=0.68$. To convert 15-minute breakdown flow rates to hourly capacity, we take an average value of 4 samples from the above random distribution. The histogram in Fig. 8 shows the probabilistic distribution of 100 lane capacity samples on link 1. The stochastic distribution of hourly capacity has a sample mean of 1837 vehicles/hour/lane and a coefficient of variation of 0.064. In Fig. 8, most of samples range from 1700 to 2100 vehicles/hour/lane, which reveals the inherent randomness of road capacity. The hourly lane capacity is multiplied by the number of lanes to generate link capacity, and the resulting average total capacity of both links is 9,298 vehicles per hour.
Measure of Effectiveness (MOE)

Mean travel time for users using ETT knowledge over different days can be represented as

\[ \bar{T}^{ETT} = \frac{1}{|d|} \sum_{d} T_{d}^{ETT} \]  \hspace{1cm} (24)

and the average travel time for ETT users on day \( d \) is

\[ T_{d}^{ETT} = \frac{\sum_{p} \sum_{i,j} (f_{p}^{ETT} \times T_{p,d}^{ETT,i,j})}{q \times (1 - \gamma)} \]  \hspace{1cm} (25)

where \( T_{p,d}^{ETT,i,j} \) is the travel time experienced by ETT users on day \( d \) along path \( p \) for OD pair \((i,j)\).

Travel time standard deviation is used to represent day-to-day travel time variability:

\[ STD^{ETT} = \sqrt{\frac{\sum{d}(T_{d}^{ETT} - \bar{T}^{ETT})^2}{(|d| - 1)}} \]  \hspace{1cm} (26)

Similarly, \( \bar{T}^{PI} \), \( \bar{T}_{d}^{PI} \) and \( STD^{ETT} \) can be calculated for PI users and \( \bar{T} \), \( \bar{T}_{d} \), STD for different classes of users.

Relative travel time improvement is defined as an indicator of the value of information:

\[ \frac{\bar{T}^{ETT} - \bar{T}^{PI}}{\bar{T}^{ETT}} \]  \hspace{1cm} (27)

This measure compares the relative difference of mean travel time savings between PI and ETT users across all days.

Sensitivity analysis

The following experiments describe the results of a sensitivity analysis for three major inputs: the total demand \( q \), the market penetration rate of PI users, and the toll value imposed on the primary route. In the baseline configuration, \( q = 8,000 \) vehicle/ hour, \( \gamma = 0.05 \) and there is no tolling on the primary routes.

MOE at varying demand levels

Fig. 9 shows the different MOEs, when the total demand level \( q \) is varied between 1,000 and 10,000 vehicles per hour. Fig. 9a shows that the average travel time dramatically increases after the total demand is raised above 4,000 vehicles per hour, which is close to the capacity of the primary route. Interestingly, Fig. 9b shows that PI users experience significantly lower travel time variability compared to ETT users.
Fig. 9. Effectiveness of information provision under stochastic capacity at varying demand levels.

MOE’s at different market penetration rates

Fig. 10 shows the sensitivity analysis results of travel time under different PI users market penetration rates. In many previous studies, the proposed models’ aim was to evaluate the effect of varying market penetration rates and identify the saturation level of market penetration of ATIS services. In this research the objective is to understand the relationship between market penetration rate and the value of information.
When the market penetration rate is below 30%, all PI users are able to switch from a congested route (typically route 1) to a less congested route (typically route 2), so that travel information provision strategies yield meaningful savings in terms of mean travel time and travel time variability (Fig. 10a, Fig. 10b). However, when the market penetration rate exceeds a certain threshold (30% in our example), a large number of PI users can take the detour, so the previously less congested route becomes crowded. Under the assumption of user equilibrium with perfect information, both routes at this point should have the same travel time so that no PI user can reduce his/her travel time by switching routes. This implies no additional benefit is available by using traveler information strategies.

Although the finding on the diminishing value of information as a function of market penetration rate is similar to a number of previous studies (for example, Yang et al., 1993; Yang, 1999), those findings are based on two fundamentally different settings. Thus, for travelers not equipped with ATIS, previous studies have considered different levels of perception errors under deterministic capacity for a single day, while this proposed approach assumes no perception error on the expected travel time of multiple days with stochastic capacity.
Fig. 10. Effectiveness of information provision under stochastic capacity with different market penetration rate.

MOE’s at different tolling rates

Fig. 11 shows the results with 5% PI users, with a static toll being imposed on route 1 to encourage route switching to route 2. The value of time is set to $15 per hour, so each dollar is equivalent to 4 minutes of travel time.

Fig. 11a shows strong bowl-shaped curves, indicating an optimal toll value of $2 in order to reduce the average (experienced) travel time to about 25.5 min for all users. This shows that the ideal system-optimal state can be approached using tolled user equilibrium patterns. Fig. 11b demonstrates another benefit of the toll strategy as it can dramatically reduce the travel time variability for all users. If the generalized travel time is considered (including both pure travel time and equivalent travel time associated with tolls), the mean generalized travel time in Fig. 11c of all the solution strategies grows gradually with increasing toll values.
Road pricing is an effective instrument for mitigating congestion. Ideally, road pricing rate is calculated based on 1) marginal social cost due to congestion (usually not perceived by the users) and 2) the private cost (perceived by the users). But applying this theory is very difficult due to the unclear relationship between the demand function and the cost function.

One key assumption in this model is that a for-pay service can advise users about the best route (with lower travel time) as shown in Fig. 12a, in spite the fact that both routes have the same disutility (travel time/VOT + toll) in Fig. 12b. Comparison of the path travel time standard deviation without tolls on route 2 and with the varying toll level on route 1 is shown in Fig. 12c. Without tolls, path travel time distributions show large standard deviations. With the toll, the path travel time distributions reveal much smaller variations depending on the toll level. Because the flow on route 1 is reduced (due to the imposed toll), travel time reliability has been decreased accordingly by charging a toll on route 1. As shown in Fig. 12d, the proposed model has the potential to hold the traffic flow within a given capacity or desirable level by applying varying tolling levels and illustrates more clearly the potential of these tolling strategies for congestion mitigation purposes when it comes to “paying” for reduced travel time.
Fig. 12. Effectiveness of information provision for different routes under stochastic capacity with varying toll.

Impact of FFTT difference and demand level on value of information

Fig. 13 shows how the free-flow travel time (FFTT) difference between the primary and second routes affects the computed value of travel information at different demand levels ranging from 5000 to 9000 veh/h. The plot demonstrates that for a fixed 5% PI market penetration rate and certain demand level (say 8000 veh/h), if both routes have similar free-flow travel times, then the value of information is not too significant. When the free-flow travel time difference increases (that is, the alternative route becomes less attractive compared to the primary route), the benefit of information provision grows steadily and reaches a maximum value. Beyond that, the value of information begins to drop, as the free-flow travel time difference is too large to generate meaningful route switching opportunities under a user equilibrium condition. By comparing the curves associated with different demand levels, the peaks of the value of information function shift to the right. Also, higher demand levels (i.e. a more congested system) yield significantly higher value of travel information at the same free-flow travel time difference.
Fig. 13. Value of information as a function of FFTT difference and demand level.

Experiments on medium-scale networks

The following numerical experiments are performed on two medium-scale network data sets, which are publicly available at a website maintained by Bar-Gera (2001). The proposed algorithm is implemented in C++ on the Windows Vista 64-bit platform and evaluated on a computer with an Intel Xeon CPU with 4 2.33 GHz processors and 9 GB memory. The proposed algorithm has been incorporated into an open-source traffic assignment package available at [https://sites.google.com/site/dtalite](https://sites.google.com/site/dtalite). To fully utilize the available parallel computing capability, all the shortest path calculations and path assignment computations for different origin zones (at steps 4 and 5) are migrated to different processors. The above parallelization scheme is implemented through an Open Multi-Processing (OpenMP) shared-memory parallel programming interface. As shown in Fig. 7, the parallelization of shortest path calculation can be also carried out for different days.

Table 2. Test network characteristics and computational performance with 30 day samples, 20 iterations and 10% PI vehicles.

<table>
<thead>
<tr>
<th></th>
<th>Anaheim, California</th>
<th>Chicago Sketch Network</th>
</tr>
</thead>
<tbody>
<tr>
<td># of nodes</td>
<td>416</td>
<td>933</td>
</tr>
<tr>
<td># of links</td>
<td>914</td>
<td>2950</td>
</tr>
<tr>
<td># of OD zones</td>
<td>38</td>
<td>387</td>
</tr>
<tr>
<td>Total OD Volume</td>
<td>104K</td>
<td>1,261K</td>
</tr>
<tr>
<td>Computational time</td>
<td>34 min</td>
<td>8 h 16 min</td>
</tr>
<tr>
<td>Average optimization Gap (min)</td>
<td>0.289</td>
<td>0.344</td>
</tr>
</tbody>
</table>

As shown in Table 2 and Fig. 14, the Anaheim, California network contains about 38 zones, and 0.1 million vehicles, and the Chicago sketch network, an aggregated representation of the Chicago region, has 387 zones with 1.2 million vehicles. Under a setting of 10% PI users, 20 assignment iterations and 30 days of random road capacity, the Anaheim network uses about 30 minutes, and the Chicago sketch network takes
about 8 hours of CPU time and 2.6G memory. There are three major factors affecting the computational complexity of the proposed algorithm: (1) the number of OD zones, (2) the number of days in the random capacity representation, and (3) the number of PI vehicles. Specifically, the first two factors are related to the number of path finding calculations, as the algorithm must find the least travel time routes using day-dependent travel time for PI vehicles originating from each origin zone. The other two factors, namely the number of days and the number of PI vehicles, jointly determine the complexity of path swapping operations, as each PI vehicle must carry and update individual paths on different days in the proposed path-based and mesoscopic representation. In addition, as steps 4 and 5 are the most computationally intensive in the proposed algorithm, additional CPU cores can also accordingly speed up the overall computational process through parallel computing.

To measure the convergence of the proposed algorithm, we use the following average optimality gap as the solution quality indicator:

\[
\text{AvgGap} = \frac{1}{D} \sum_{i,j} q_{i,j} \sum_{d} \sum_{i} \sum_{j} \sum_{p} \left[ f_{p,d}^{PI,i,j} \times \left( U_{p,d}^{i,j} - \pi_d^{i,j} \right) + f_{p}^{ETT,i,j} \times \left( \overline{U}_{p}^{i,j} - \overline{\pi}^{i,j} \right) \right].
\]  

Fig. 15 compares the convergence of two algorithms in the Chicago sketch network: (i) the proposed route-swapping algorithm that uses the swapping ratio in Eq. (22), (ii) the algorithm uses the conventional MSA method with a fixed step size of 1/(n+1), where \( n \) is the iteration counter. Both algorithms can steadily reduce the optimality gap toward zero, and the average gap measures reach within 0.5 minutes per vehicle within the first 10 iterations. In comparison, the proposed route swapping rule is able to more quickly decrease the optimality gap within first 5-7 iterations. Shown in Table 2, the two test networks have quite small optimality gaps at iteration 20, namely 0.289 and 0.344 minutes. To reduce the overall computational efforts, one might consider using fewer iterations as long as the solution quality reaches an acceptable level.

Fig. 14. Chicago sketch network (left) and Anaheim, California network (right)
Fig. 15. Convergence patterns of route-swapping and MSA rules on Chicago sketch network

The original data sets use the BPR function to describe travel time performance, and a single valued mean capacity is specified for each link. To generate random road capacity samples, we use the pre-breakdown headway distribution in Eq. (23) to generate multiple samples of 15-min pre-breakdown capacity first. To approximate the peak hour capacity values used in the BPR function, we evaluate the impact of two alternative schemes:
(i) Possible multiple congestion periods within a peak hour, so we use the average of 4 pre-breakdown capacity values as the peak hour capacity, leading to a Coefficient of Variation (CV) = 6.4%
(ii) Single congestion period, where a single value of 15-min pre-breakdown capacity becomes the dominating factor for the whole peak hour, leading to CV = 12.8%

Table 3. Value of traveler information under different peak-hour capacity approximation schemes.

<table>
<thead>
<tr>
<th>Peak-hour Capacity Approximation Scheme</th>
<th>Anaheim network</th>
<th>Chicago Sketch Network</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ETT Travel Time</td>
<td>PI Travel Time</td>
</tr>
<tr>
<td>Average Pre-breakdown Capacity with CV = 0.064</td>
<td>12.903</td>
<td>12.864</td>
</tr>
<tr>
<td>Single-valued Pre-breakdown Capacity with CV = 0.128</td>
<td>13.233</td>
<td>13.157</td>
</tr>
</tbody>
</table>

As shown in Table 3, the single valued pre-breakdown capacity implies larger variations compared to the average (aggregated) pre-breakdown capacity, so it slightly increases the average travel time for PI and ETT travelers on both networks. However, according to the above experimental results, the travel time savings obtainable for PI users seem to be not too significant even under the large link travel time variations. Similarly, for the hypothetical network in Fig. 2, the capacity variations also only lead to less than 1 minute travel time savings due to traveler information provision, shown in Fig. 10-(a).
To fully understand the benefit of traffic information provision, an analyst needs to better characterize travel time dynamics/variability, which is caused by a wide range of recurring and non-recurring delay sources, such as incidents, work zones, and random fluctuations in road capacity. Although the proposed framework allows and is naturally suited to consider any given random capacity distributions with a multi-day or sample-based representation scheme, the calibrated capacity distribution used in our study in fact only focuses on “normal” random capacity perturbations, while the “outliers” in the capacity distribution due to nonrecurring events such as incidents, work zone, severe weather, are not explicitly modeled in the given capacity distribution and should be also integrated in the future research to fully account for the benefits of traveler information provision strategies under random capacity breakdowns and “unplanned” events.

7. Conclusions

In this paper, a novel non-linear optimization-based analysis method is proposed along with related modeling components pertaining to stochastic capacity, travel time performance functions and different degrees of traveler knowledge in an ATIS environment. Within a multi-day analysis framework, this proposed method categorizes commuters into two classes: (1) travelers with access to perfect traffic information every day, and (2) travelers with some degree of knowledge of average traffic conditions across different days. Within a gap function framework (for describing the user equilibrium under different information availability), a mathematical programming model is formulated to describe the route choice behavior of the perfect information (PI) and expected travel time (ETT) user classes under stochastic day-dependent travel time. The model was applied to a simple 2-link corridor and two medium-scale networks to illustrate the effectiveness of traveler information under stochastic capacity. Some of the key findings are summarized below.

1. Under stochastic link capacity, users equipped with ATIS can reduce both their mean travel time and travel time variability across different days, compared to travelers who rely only on knowledge of average traffic conditions.

2. Equipping a small percentage of users with access to travel information can help the system better balance flow between congested and uncongested routes, and fully utilize available unused capacity.

3. Better system-wide benefits regarding travel time saving and reliability are achieved compared to the no information or limited information cases until reaching a saturated market penetration rate.

4. An appropriate tolling or real-time information service scheme may encourage travelers to switch from a congested to a less congested route under stochastic link capacity conditions, and it can improve both travel time and its reliability. On the other hand, the generalized travel time for users is increased due to an inclusion of equivalent travel time and user cost associated with charged toll.

Quantifying benefits of traveler information provision strategies in a stochastic environment places a great need for rigorous formulations and practical solution procedures for the traffic network assignment problem. Given a wide range of travel time variability sources, it is desirable to further enhance the proposed model to systemically evaluate the value of information and reliability associated with stochastic demand fluctuations and different levels of information quality, under both recurring and nonrecurring congestion conditions. The computational challenges introduced by the proposed method stem from the sampling-based representation of stochastic capacity distributions. Our future research plans to use variance reduction techniques, such as importance sample, to reduce the required sample size, and apply distributed computing techniques, such as cloud computing, to improve the computational efficiency within a non-shared memory environment.

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