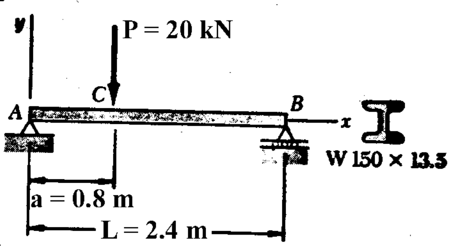
Example Illustrating Calculation of Elastic Curve and Vertical Deflection of a Beam

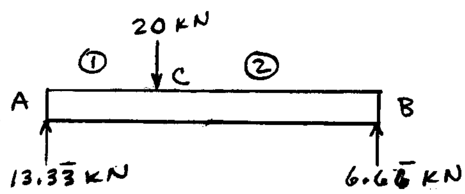
Given: The simply supported beam shown below; $E = 200$ GPa.

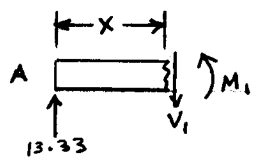


Required: Determine (a) the equations of the elastic curve for the beam, and (b) the deflection at Pt. C.

Assumptions: Deflections and angles of curvature are small.

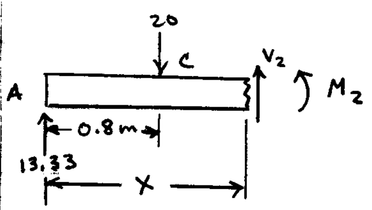
Solution: Find equations for M_1 (left of Pt. C) and M_2 (right of Pt. C).





$$\Sigma M_A = -13.33x + M_1 = 0$$

$$M_1 = +13.33x$$

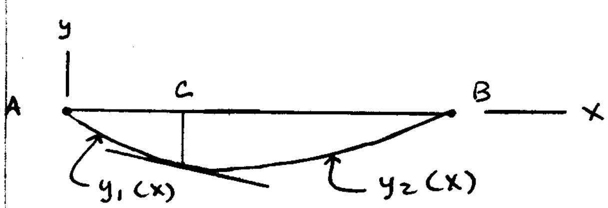


$$\Sigma F_y = +13.33 - 20 + V_2 = 0 \Rightarrow V_2 = 6.6\bar{6}\text{ kN}$$

$$\Sigma M_A = -(0.8)(20) + 6.6\bar{6}x + M_2 = 0$$

$$M_2 = +16 - 6.6\bar{6}x$$

We need to write an equation for $y(x)$ for portions ① and ②:



Boundary Conditions

At $x = 0$: $y_1 = 0$

At $x = 0.8$ m: $y_1 = y_2$, $\dfrac{dy_1}{dx} = \dfrac{dy_2}{dx}$

At $x = 2.4$ m: $y_2 = 0$

Equations

$$EI\frac{d^2y_1}{dx^2} = M_1 = +13.3\bar{3}x$$

Integrating: $$EI\frac{dy_1}{dx} = +6.6\bar{6}x^2 + C_1 \qquad ①$$

Integrating: $$EI\,y_1 = +2.2\bar{2}x^3 + C_1x + C_2 \qquad ②$$

$$EI\frac{d^2y_2}{dx^2} = M_2 = +16 - 6.6\bar{6}x$$

Integrating: $EI\frac{dy_2}{dx} = +16x - 3.3\bar{3}x^2 + C_3$ (3)

Integrating: $EI\,y_2 = +8x^2 - 1.1\bar{1}x^3 + C_3x + C_4$ (4)

Determination of Constants

Determination of y at $x = 0.8$ m

From Appendix C, p. 753: $I_x = 6.87\times10^6$ mm^4 $= 6.87\times10^{-6}$ m^4

$E = 200$ GPa $= 200\times10^6$ kPa $= 200\times10^6$ kN/m^2

y_1 at $x = 0.8$ m: $EI\,y_1 = +2.2\bar{2}(0.8)^3 - 7.1\bar{1}(0.8) + 0 = -4.55\bar{1}$

$$y_1 = \frac{-4.551}{(200\times10^6)(6.87\times10^{-6})} = -0.00331\text{ m} = -3.31\text{ mm}$$

y_2 at $x = 0.8$m: $EI\,y_2 = +8(0.8)^2 - 1.1\bar{1}(0.8)^3 - 13.5\bar{1}(0.8) + 1.70\bar{6}$

$EI\,y_2 = -4.55\bar{1}$ checks

Summary of Answers

(a) For $x \le 0.8$m: $EI\frac{dy}{dx} = +6.6\bar{6}x^2 - 7.11$ (b) $y_c = -3.31$mm

For $x \ge 0.8$m: $EI\frac{dy}{dx} = +16x - 3.3\bar{3}x^2 - 13.5\bar{1}$