LIMIT EQUILIBRIUM AND CONTINUUM MECHANICS-BASED
NUMERICAL METHODS FOR ANALYZING STABILITY OF MSE WALLS

Jie Han¹ (Member, ASCE) and Dov Leshchinsky² (Member, ASCE)

ABSTRACT
Limit equilibrium (LE) methods have been commonly used to analyze stability of geosynthetic-reinforced slopes. LE methods assume a potential slip surface, the soils along this slip surface providing shear resistance, and geosynthetic reinforcement providing tensile forces and resisting moments. Continuum mechanics-based numerical method has become increasingly used in recent years for slope stability analysis. Continuum mechanics-based numerical method assumes a reduction of soil strength by a factor to reach a critical state prior to failure. Both methods yield factors of safety of the system. This paper presents a study to investigate the stability of MSE walls (vertical or 20° batter) using LE and numerical methods. The comparisons of the critical slip surface and the factor of safety are made when the predicted factor of safety using the LE approach is equal to 1.0. It is concluded that properly adopted LE approach can be used to analyze the stability of MSE walls.

Keywords: Continuum mechanics, factor of safety, limit equilibrium, walls

INTRODUCTION
Limit equilibrium methods have been used for decades to safely design major geotechnical structures. Bishop’s simplified method, utilizing a circular arc slip surface, is probably the most popular limit equilibrium method. Although Bishop’s method is not rigorous in a sense that it does not satisfy horizontal force limit equilibrium, it is simple to apply and, in many practical problems, it yields results close to rigorous limit equilibrium methods. In this work, Bishop’s simplified method was modified to include reinforcement as a horizontal force intersecting the slip circle. Unlike the approach proposed by Elias and Christopher (1997) which considers the reinforcement contribution as a pure moment, the modified approach considers the

¹ Corresponding author, Assistant Professor, Ph.D., PE, Dept. of Civil Engineering, Widener University, One University Place, Chester, PA 19013, USA, e-mail: jxh0305@mail.widener.edu
² Professor, Ph.D., Dept. of Civil and Environmental Engineering, University of Delaware, Newark, DE 19716, USA, e-mail: dov@ce.udel.edu
reinforcement producing a tensile force to generate a resisting moment as well as affect the normal force on the slip surface thus affecting shear resistance. This modified formulation is consistent with the original formulation by Bishop (1955). The mobilized reinforcement strength at its intersection with the slip circle depends on its long-term strength, its rear-end pullout capacity (or connection strength), and Bishop’s factor of safety. The analysis assumes that when the soil and reinforcement strengths are reduced by the factor of safety, a limit equilibrium state is achieved (i.e., the system is at the verge of failure), meaning that under this state, the soil and reinforcement mobilize their respective strengths simultaneously. Obviously, limit equilibrium is physically meaningful only at the verge of failure, regardless whether reinforcement is invoked. However, if one can define or predict this state, then in design one can assure that a certain minimum margin of safety (i.e., factor of safety) against that state exists. That is, one can ensure that the existing soils and reinforcement are stronger by a certain margin than the value rendering the verge of failure. The Bishop option in ReSSA(2.0) software, developed by ADAMA Engineering (2002), was utilized to generate the results in this work. Detailed discussion on this software can also be found in the literature by Leshchinsky (2002).

CONTINUUM MECHANICS-BASED NUMERICAL METHOD

The finite difference program (FLAC 2D Version 4.0, developed by the Itasca Consulting Group, Inc.), which is based on continuum mechanics, was adopted in this study for numerical analyses of the stability of MSE walls. A shear strength reduction technique was adopted in this program to solve for a factor of safety of slope stability. Dawson et al. (1999) exhibited the use of the shear strength reduction technique in this finite difference program and verified numerical results with limit equilibrium results for simple slopes. In this technique, a series of trial factors of safety are used to adjust the cohesion, $c$ and the friction angle, $\phi$, of soil as follows:

$$c_{\text{trial}} = \frac{1}{FS_{\text{trial}}}c$$

(1)

$$\phi_{\text{trial}} = \arctan \left( \frac{1}{FS_{\text{trial}}} \tan \phi \right)$$

(2)

Adjusted cohesion and friction angle of soil layers are re-inputted in the model for equilibrium analysis. The factor of safety is sought when the specific adjusted cohesion and friction angle make the slope become instability from a verge stable condition or verge stable from an unstable condition.

Cundall (2002) compared the characteristics of numerical solutions and limit equilibrium methods in solving the factor of safety of slopes and concluded that continuum mechanics-based numerical methods have the following advantages: (1) No pre-defined slip surface is needed; (2) The slip surface can be of any shape; (3) Multiple failure surfaces are possible; (4) No statical
assumptions are needed; (5) Structures (such as footings, tunnels, etc.) and/or structural elements (such as beams, cables, etc.) and interfaces can be included without concern about compatibility; and (6) Kinematics is satisfied. However, a complicated and large size problem may require significant computation time for numerical methods. The inclusion of structural elements and interfaces may create numerical instability leading to questionable solutions. Some specific searches are difficult to perform (for example, surficial slope instability needs to be prevented in order to study the deep-seated slope stability). Localized and inconsequential failures, which may not be of interest to the study (e.g., locally overstressed soil), may mislead the investigation. In short, while continuum mechanics-based methods rigorously satisfy equilibrium and boundary conditions, it generally requires an experienced analyst in order to properly use it. However, being of a higher hierarchy in mechanics and if properly used, it can serve effectively to substantiate the validity of a simpler limit equilibrium approach which uses an a priori assumed failure mechanisms and which fails to rigorously satisfy equilibrium (e.g., Bishop Method). That is, it can justify the use of a simpler and more tangible approach.

Considering the rigor of the continuum mechanics-based numerical method, the program FLAC was adopted in this study to evaluate MSE walls. The computed factors of safety and their respective slip surfaces were compared with those obtained using Bishop Method as produced by program ReSSA(2.0). The ability of FLAC to assess slope stability in a similar manner to limit equilibrium (i.e., same definition of factor of safety) makes such comparison meaningful.

MODEL AND METHOD OF APPROACH

The geometry and material properties of the model used in this study are presented in Figure 1 and Table 1. Since the factor of safety is determined based on a state of yield, or verge of failure, it is insensitive to the selected elastic parameters: Young’s modulus (E) and Poisson’s ratio (ν) when using FLAC. If the system contains soils with largely different elastic parameters, it will take longer time to solve for the factor of safety; however, the effects on this factor would be small since it depends mainly on Mohr-Coulomb strength parameters. Hence, constant values of E = 40MPa and ν = 0.25 were used in FLAC. A small value of cohesion equal to 2.5kPa was used for blocks to prevent possible local failure of the block facing thus enabling one to focus on global failure modes. The last two blocks were assumed to have cohesion of 0.01kPa to ensure the exit of the slip surface through the toe. The MSE wall with a batter of 0° (vertical) or 20° was investigated in this study. The 20° batter was selected based on the fact that the MSE wall with a batter ranging from 0° to 20° is typically designed using the wall method (for example, AASHTO, 1998). In addition, the MSE structure with a 45° batter has been investigated in the previous study by Han et al. (2002). Mohr-Coulomb failure criterion was used for strength between stacked blocks, for the reinforced and retained fill, and for the foundation soil. The bond strength between reinforcement and reinforced fill was assumed equal to 80% the fill strength, same as in the limit equilibrium analysis when considering pullout resistance.
Fig. 1. Model for limit equilibrium and numerical analyses

TABLE 1. Material Properties Used in the Analyses

<table>
<thead>
<tr>
<th>Materials</th>
<th>Blocks</th>
<th>Reinforced and retaining fill</th>
<th>Foundation soil</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Properties</td>
<td>$\gamma = 18 \text{kN/m}^3$, $c = 2.5 \text{kPa}$, $\phi = 34^0$</td>
<td>$\gamma = 18 \text{kN/m}^3$, $c = 0 \text{kPa}$, $\phi = 34^0$</td>
<td>$\gamma = 18 \text{kN/m}^3$, $c = 10 \text{kPa}$, $\phi = 34^0$</td>
<td>$T_a = 11.1 \text{kN/m}$ (vertical wall) or $T_a = 6.2 \text{kN/m}$ ($20^\circ$ batter), $C_i = 0.8$</td>
</tr>
</tbody>
</table>

$\gamma$ = unit weight, $c$ = cohesion, $\phi$ = friction angle, $T_a$ = design tensile strength of reinforcement, and $C_i$ = interaction coefficient of reinforcement and soil.

In this work, the required strength of reinforcement was first determined using ReSSA(2.0). The strength value was varied iteratively while holding all other variables constant until the computed factor of safety of the wall system was equal to 1.0. When the factor of safety is 1.0, the system is at the verge of failure and thus it corresponds to a physically meaningful state of limit equilibrium. This tensile strength of the reinforcement was then input into FLAC analysis to compute the factor of safety for the same system. If, for the same problem, FLAC produces a factor of safety of approximately 1.0 (and a shear zone that is similar to the trace of the limit equilibrium trace of critical surface), the ‘benchmark’ of the limit equilibrium state produced by ReSSA(2.0) is considered equivalent to FLAC. Consequently, this benchmark is theoretically confirmed by using a higher hierarchy, more rigorous, mechanics. It should be noted that in limit equilibrium analysis, it is convenient to apply the factor of safety also to the reinforcement strength (unlike the version of FLAC used which applies it only to the soil strength); however, selecting a factor of safety of 1.0 in limit equilibrium makes this difference in definition inconsequential in the context of this study provided the resultant factors of safety are 1.0.
ANALYSES AND DISCUSSION

Potential Slip Surface

Potential slip surfaces are assumed in the LE analysis to calculate their corresponding factors of safety. Figure 2(a) presents the spatial distribution of the safety factors for the vertical MSE wall from ReSSA (2.0) program, which provides a diagnosis of the stability of the analyzed wall for the specified search domain using Bishop’s analysis. This technique is included based on the “safety map” concept proposed by Baker and Leshchinsky (2001). It is clearly shown that a marginal factor of safety (FS = 1.0-1.1) zone exists within the reinforced fill.

Instead of spatial distribution of factors of safety, however, FLAC program defines the plasticity zone when any location is at yield due to shear. Figure 2(b) presents the result of plasticity zone from the FLAC analysis. This plasticity zone covers the region defined by the factors of safety less than 1.3 as shown in Figure 2(a). It should be pointed out that the geosynthetic reinforcement does not necessarily reach the maximum tensile strength at the location the soil has yielded. As indicated in the previous study by Han et al. (2002), the shear strains developed in the reinforced slopes depend on the tensile stiffness of geosynthetics. The geosynthetics with higher tensile stiffness minimize the shear strains developed in the reinforced slopes because the geosynthetics with higher stiffness mobilize their tensile forces more quickly so that the soil carries less loads. For the same reason, the plasticity zone shown in Figure 2(b) also depends on the tensile stiffness of geosynthetics.

Figure 3 presented a similar comparison between the spatial distribution of factors of safety by ReSSA and the plasticity by FLAC for an MSE wall with 20° batter. In this comparison, the region with a factor of safety of 1.0 to 1.1 matches well with the plasticity zone. The plasticity zone in Figure 3(b) is much narrower than that in Figure 2(b). This implies less soil yielding.
Critical Slip Surface and Minimal Factor of Safety

The critical slip surface in LE analysis is determined based on the minimal factor of safety in the evaluation of the factors of safety corresponding to many potential slip surfaces. The critical slip surface determined using Bishop’s method in ReSSA (2.0) program for the vertical MSE wall is presented in Figure 4. This critical slip surface corresponds to the factor of safety of 1.00. As shown in Figure 4(a), the slip surface starts from the toe and follows the Rankine’s slip plane (the dash line) up to 1/3 height of the wall then bends towards the wall facing. This outcome results from the assumptions of Bishop’s method. Bishop’s method, by solving the moment equilibrium equation, attempts to minimize the effects of the upper layers. The center of the critical circle is at the same elevation as the crest in this case, which makes the moment contribution of upper layers minimal. In addition, ReSSA program (2.0) also calculates the imbalanced horizontal force. In this case, the imbalanced horizontal force is about 20% of the total weight of slices. In other words, the horizontal force equilibrium is grossly violated.

The contour of the maximum shear strain rate determined by FLAC is presented in Figure 4(b). It is shown that the maximum shear strain rate, which indicates the critical slip surface, develops along the Rankine slip plane (the dash line). However, this result is different from that by Bishop’s method in ReSSA (2.0) program as shown in Figure 4(a). In spite of this difference, the calculated factor of safety by the LE method in ReSSA (2.0) and that by the continuum mechanics-based numerical method in FLAC are close (1.00 versus 1.04).

A similar comparison between the critical slip surface by ReSSA and the maximum shear strain rate contour by FLAC for the MSE wall with 20° batter is presented in Figure 5. Both results indicate that the critical slip surface is circular. However, the maximum shear strain rate contour indicates that the critical slip surface is a little bit deeper. In this case, the imbalanced horizontal force calculated by ReSSA is about 5% of the total weight of slices, which is much
less than that for the vertical wall. In the previous study by Han et al. (2002) for a 45° batter MSE structure (slope), the critical slip surface predicted by Bishop’s method in ReSSA and the maximum shear strain rate contour by FLAC matched very well. This implies that the location of the critical slip surface predicted by LE method (Bishop’s method) gets closer to that predicted by the continuum mechanics-based numerical method when the batter of the MSE structure increases from 0° to 45°. As shown in Figure 5, the calculated factor of safety for the MSE wall with 20° batter by the LE method in ReSSA (2.0) and that by the continuum mechanics-based numerical method in FLAC are close (1.00 versus 0.96).

![Critical slip surfaces and minimal factors of safety (vertical wall)](image)

(a) Critical slip surface and FS by ReSSA    (b) Maximum shear strain rate and FS by FLAC

**Fig. 4. Critical slip surfaces and minimal factors of safety (vertical wall)**

![Critical slip surfaces and minimal factors of safety (20° batter MSE wall)](image)

(a) Critical slip surface and FS by ReSSA    (b) Maximum shear strain rate and FS by FLAC

**Fig. 5. Critical slip surfaces and minimal factors of safety (20° batter MSE wall)**
CONCLUSIONS

In this study, limit equilibrium (LE) and continuum mechanics-based numerical methods were used to investigate the stability of MSE walls. This investigation indicates that there is a difference in the location of the critical slip surface predicted by the LE method and the numerical method. The difference becomes less when the batter of the MSE wall increases. In spite of the difference in the critical slip surface, the factors of safety computed by the LE method and the numerical method are very close. Since the factor of safety is the key to designing MSE walls in terms of stability, properly adopted LE approach can be used to fulfill this purpose.

REFERENCES

ADAMA Engineering, Inc. (2002). ReSSA Version 2.0. Newark, Delaware, USA.