Critical-state soil mechanics is an effective stress framework describing mechanical soil response.

In its simplest form here, we consider only shear-induced loading.

We tie together two well-known concepts: (1) one-dimensional consolidation behavior, represented by (e-log \( \sigma_v' \)) curves; and (2) shear stress-vs. normal stress (\( \tau-\sigma_v' \)) from direct shear box or simple shearing.

Herein, only the bare essence of CSSM concepts are presented, sufficient to describe strength & compressibility response.
In an attempt to advance soil testing techniques, Kenneth Harry Roscoe of Cambridge University, in the late forties and early fifties, developed a simple shear apparatus in which his successive students attempted to study the changes in conditions in the shear zone both in sand and in clay soils. In 1958 a study of the yielding of soil based on some Cambridge data of the simple shear apparatus tests, and on much more extensive data of triaxial tests at Imperial College, London, led to the publication of the critical state concept (Roscoe, Schofield & Wroth 1958). Subsequent to this 1958 paper, concepts of plasticity were introduced by Schofield and published later in a classic text book (Schofield & Wroth 1968).

Direct Shear Test Results

Slow Direct Shear Tests on Triassic Clay, NC

Slow Direct Shear Tests on Triassic Clay, Raleigh, NC

Strength Parameters:
\( c' = 0; \phi' = 26.1^\circ \)

\( 0.491 = \tan \phi' \)

Shear Strength Theory

Thursday, March 11, 2010
11:43 AM
The **critical state concept** states that soils and other granular materials, if continuously distorted (sheared) until they flow as a frictional fluid, will come into a well-defined critical state. At the onset of the critical state, shear distortions occur without any further changes in mean effective stress or deviatoric stress or void ratio. The void ratio at the critical state is called the critical state void ratio.

One-Dimensional Consolidation

Overconsolidation Ratio, $OCR = 3$

$C_r = 0.04$

Effective Vertical Stress, $svo' (kPa)$

Over consol idation Ratio, $OCR = 3$

$\sigma_v' = 300 \text{ kPa}$

$\sigma_p' = 900 \text{ kPa}$

In some textbooks the effective vertical stress is replaced with the mean effective stress, which is:

$$\sigma_m' = \frac{(\sigma_{v1}' + \sigma_{v2}' + \sigma_{v3}')}{3}$$
CSSM Premise:
“All stress paths fail on the critical state line (CSL)”

The above statement means that the critical state line forms an envelope that defines the failure state of soil. This failure or critical state is a function of the state of stress (vertical or mean effective stress and shear stress) and the void ratio.
STRESS PATH No. 1
NC Drained Soil
Given: $e_0$, $\sigma'_v$, NC (OCR=1)

Volume Change is Contractive:
$\varepsilon_{vol} = \Delta e/(1+e_0) < 0$

The above stress path is straight because there is no excess pore pressure generated during shear because the test is drained and the applied vertical stress is not changing during the direct shear mode of failure.
Stress Path for Normally Consolidated Undrained Soil (DSS)

STRESS PATH No. 2
NC Undrained Soil

Given: \( e_0, \sigma_{vo}', \text{ NC (OCR}=1) \)

\[ +\Delta u = \text{Positive Excess Porewater Pressures} \]

Note that there is no change in void ratio in the above consolidation plots because the test is undrained.
Effect of Increasing effective vertical stress on undrained strength

Note: All NC undrained stress paths are parallel to each other, thus:

\[ \frac{s_u}{\sigma_v'} = \text{constant} \]

DSS: \[ \frac{s_u}{\sigma_v'_{NC}} = \frac{1}{2} \sin \phi' \]
Overconsolidated States:

$e_0$, $\sigma_{vo}'$, and $OCR = \sigma_p'/\sigma_{vo}'$

where $\sigma_p' = \sigma_{vmax}' = P_c' =$ preconsolidation stress;
$OCR =$ overconsolidation ratio
Stress Path for Undrained Over-consolidated Soil (DSS)

Stress Path No. 3
Undrained OC Soil:
e₀, σvo', and OCR
Stress Path: ΔV/V₀ = 0
Negative Excess Δu
The behavior of the soil is elastic until the state of stress on the soil reaches the yield surface. After that, the soil behaves in a plastic manner and undergoes both elastic and plastic strains. The yield surface will expand as the soil dilates or strain hardens: it will contract as the soil contracts or strain softens until the critical state is reached. At this point, this yield surface no longer changes and is known as the yield surface at critical state.
Initial state: $e_0$, $s_{vo}'$, and OCR = $s_p'/s_{vo}'$

Soil constants: $f'$, $C_c$, and $C_s$ (L = 1-$C_s/C_c$)

Using effective stresses, CSSM addresses:

- NC and OC behavior
- Undrained vs. Drained (and other paths)
- Positive vs. negative porewater pressures
- Volume changes (contractive vs. dilative)
- $s_u/s_{vo}' = \frac{1}{2} \sin f' \text{OCR}$ where $L = 1-C_s/C_c$
- Yield surface represents 3-d preconsolidation